NEURAL NETWORKS WITH HYSTERESIS TYPE OF NONLINEARITY EXHIBIT GLOBAL OPTIMIZATION PROPERTY

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Abstract

The minimization of quadratic forms over discrete sets plays a central role in many areas of applications like communication and control theory and pattern recognition. That is why, among the primary interest of neural network (NN) research, the global optimization problem has received distinctive attention. Nevertheless, most of the commonly implemented neural networks either fail to achieve the global optimum like the Hopfield model [3,4], or the necessary amount of computation needed by the optimization is large and time consuming (Boltzmann machine [1]). As a result, the aim of this paper is to propose a new type of NN capable of quick minimization of quadratic forms. Since the fast optimization of quadratic forms is very much required in communication theory, one of the most promising applications of the new algorithm would be the optimal detection procedure of Gaussian and linearly distorted channels.

1 INTRODUCTION

The primary importance of the minimization of quadratic forms over discrete sets has much stimulated the neural network research, as well as many other fields of applied mathematics. Among many others, this was one of the main motivations leading to the intensive research on the so-called Hopfield model.

However, the failure of the Hopfield net (HN) as global optimizer is not surprising, owing to the difference between global optimization and associative memories which can be easily understood by the following reasoning:

In the case of an associative memory the network has to possess a large number of steady states, corresponding to the stored memory objects (for maximizing the capacity [3,2]), and local stability on these fixed points. These properties, however, are apparently useless for a 'global optimizer' when only one absolute stable steady state associated with the global minimum is required.

However, the Hopfield model shows some properties (e.g. fast convergence, simple hardware implementation) which are worthwhile to be preserved in the case of a global optimizer, as well.
Thus, the aim of this paper is to introduce a new type of NN for solving global optimization tasks related to quadratic forms, preserving the advantages of the classical methods.

The novelty of the proposed model lies in the application of a negative hysteresis type of non-linearity as a threshold function of the neurons. This negative hysteresis enables the network to get rid of local minima and to recover quickly the global optimum, acting just the opposite way as the positive hysteresis introduced by H. Yanai and Y. Sawada [6] in order to raise the number of local optima for increasing the capacity in a case of an associative memory. Owing to the primary interest of quadratic forms in detection theory, this structure can be used as an optimal detector in communication systems, where quick and simple detection algorithms are needed.

However, the proposed network can be applied in many other problems which correspond to the optimization on quadratic forms.

Most communication systems corrupted by additive Gaussian noise and channel distortion can be modeled by the following discrete time system (Figure 1):

\[ Z_k = f(v_k, \ldots, v_{k-N}) \]

\( Z_k \): binary independent identically distributed random variables
\( h_k \): discrete impulse response of the channel
\( v_k \): Gaussian noise sequence \( E v_k = 0, E v_k v_l = K_k l = K_{k-l} \) and \( E v_k^2 = \sigma^2 \)
\( y_k \): a sample of \( Z_k \)
\( \hat{y}_k \): a sample of \( \hat{Z}_k \)
\( N \): the length of the transmitted sequence

Figure 1: Digital communication system with channel distortion and noise.

The problem posed by the detection of noisy and distorted sequences can be summarized as follows:

What is the optimal \( \hat{Z}_k = f(v_k, \ldots, v_{k-N}) \) mapping (detector) on the received sequence which minimizes the error probability defined as \( P_E := P \left( \bigcup_{k=1}^{N} \{ Z_k \neq \hat{Z}_k \} \right) \)?

As has already been proven [5], in the case of Gaussian noise the optimal detection leads to the