A Note on Broadcasting with Linearly Bounded Transmission Faults in Constant Degree Networks

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Abstract. We consider the problem of broadcasting in the presence of linearly bounded number of transient faults. For some fixed 0 < α < 1 we assume that at most αi faulty transmissions can occur during the first i time units of the broadcasting algorithm execution, for every natural i. In a unit of time every node can communicate with at most one neighbor. We prove that some bounded degree networks are robust with respect to linearly bounded transmission faults for any 0 < α < 1, and give examples of such networks with logarithmic broadcast time, thus solving the open problem from [3].

1 Introduction

Broadcasting is one of the most fundamental tasks in network communication. Its aim is to transmit the information originally held in one node of the network (called the source) to all other nodes.

Recently, fault-tolerant broadcasting has become an extensively studied domain (for a survey see [4]). The goal is to effectively accomplish the communication task in spite of failure of some network components. The wide range of features necessary to fully describe the model (such as the communication mode, the type, number and duration of faults, etc.) offers a vast area for research.

Let us begin with the detailed description of the model considered in this paper. The network is synchronous — we assume the existence of a global clock. In a single time unit a node can communicate with at most one of its neighbors. During a fault-free transmission information can pass in both directions; a faulty transmission has no effect. Our algorithms are non-adaptive, i.e. all transmissions are scheduled in advance and do not depend on execution history. We consider the linearly bounded fault model. Given a constant 0 < α < 1 we assume that at most αi arbitrarily placed transient faulty transmissions can occur during the first i time units of the communication process.

For a fixed parameter 0 < α < 1, a given network \( \mathcal{N} \), and a source node s, a broadcasting algorithm is called α-safe if it broadcasts information from the source s to all nodes of \( \mathcal{N} \), whenever the number of faulty transmissions during the algorithm execution satisfies the above assumption with parameter α. A network in which α-safe broadcasting can be accomplished in time linear in its fault-free broadcasting time, for any source s and constant 0 < α < 1, is called robust with respect to linearly bounded transmission faults.

* Supported by the grant KBN 8T11C01208.
(It might seem more natural to let the number of faults be proportional to the number of messages sent rather than to the number of time units. However, it can be shown that in this case a non-adaptive $\alpha$-safe broadcasting algorithm requires an exponential number of messages, even in a complete network, for any $0 < \alpha < 1$. Thus, this variation of the fault model is not interesting.)

The linearly bounded fault model described above and previously used in [5, 1] in the context of searching with errors, has been studied by Gasieniec and Pelc [3]. One of their results was the proof that the hypercube is robust with respect to linearly bounded transmission faults. They stated the following question as open: is there a bounded degree network for which $\alpha$-safe broadcasting time is logarithmic for any constant $0 < \alpha < 1$? Our main result is the affirmative answer to this question.

The rest of this paper is organized as follows. In section 2 we present a general broadcasting algorithm and prove that it works fast in the linearly bounded fault model under some assumptions about the network connectivity. In section 3 we apply this general method to show that constant degree multidimensional tori and wrap-around butterflies are robust with respect to linearly bounded transmission faults. Section 4 contains conclusions.

2 Broadcasting through perfect matchings

Let $G = (V, E)$ be the underlying graph of our network and assume that the set of its edges $E$ can be partitioned into pairwise disjoint perfect matchings $M_1, \ldots, M_d$. Consider the following algorithm.

**Algorithm** BROADCAST($t, M_1, \ldots, M_d$)

repeat $t$ times
   for $i = 1..d$ do
      for $(v, w) \in M_i$ in parallel do
         nodes $v$ and $w$ exchange information

   Its execution time is clearly $td$.

**Lemma 1.** Let $s, v_1, \ldots, v_d \in V$ (the nodes $v_1, \ldots, v_d$ are not necessarily different) and assume that there exist edge-disjoint paths between $s$ and $v_i$ for $i = 1, \ldots, d$, each path of length at most $D$. If the node $s$ is initially informed and there are at most $k$ transmission faults during the execution of the algorithm BROADCAST then after $dD + k$ time units at least one of the nodes $v_1, \ldots, v_d$ becomes informed.

**Proof.** Each transmission fault on a single path delays the information flow on this path for at most $d$ time units. Since there are at most $k$ faults on $d$ paths, there is a path with at most $\lfloor k/d \rfloor$ faults. The information reaches its end in at most $d(D + \lfloor k/d \rfloor) \leq dD + k$ time units. $\square$