Abstract

We present a way to construct an infinite family of linear codes, with \( d=3 \), from a particular type of strongly regular graph.

Using the incidence matrix and taking a tree of any nondirected and connected graph \( G(V,E) \) we can obtain the matrix of fundamental circuits and the one for fundamental cuts. These are respectively the generator and parity check matrices of a linear code.

We are interested in the construction of triangular strongly regular graphs, named \( T(m) \). From \( T(m) \) and some results on the girth and the valency of a strongly regular graph \( G \), for any integer \( m \geq 4 \), we obtain a linear code \( C(T(m)) \) with parameters: \( n=(m(m-1)(m-2))/2 \), \( k=(m(m-1)(m-3)+2)/2 \), \( d=3 \) and \( A_i=A_{n-i} \), \( A_i \) being the number of codewords of weight \( i \) in the code.

Moreover we give some properties of its codewords set and its orthogonal one. Finally, an alternative method using lattice graphs \( L_2 \) is proposed.
1. Graphs: definitions and basic properties

A graph \( G(V,E) \) is formed by a vertex-set \( V \) and an edge-set \( E \), where every edge is a subset of cardinality 2 of \( V \). The graph is regular if every vertex appears in the same number of edges.

Let \( G(V,E) \) be a connected graph with \(|V|=n\) and \(|E|=m\). A path is a sequence of adjacent edges, and a tree of \( G \) is any subset \( T \) of \( E \) with \(|T|=n-1\) that establishes one and only one path between every pair of vertices in \( G \).

For a tree \( T \) of \( G \) any \( e \in E/e \not\in T \) is called a chord of \( T \).

Let \( Y_{n,m} \) be the \((0-1)\)-incidence-matrix of \( G \) where

\[
Y(i,j) = \begin{cases} 
1 & \text{if } e_j \text{ links } v_i \\
0 & \text{otherwise.}
\end{cases}
\]

A circuit is a path that begins and finishes at the same vertex. The \( m-(n-1) \) circuits obtained by the addition of any chord to \( T \) are called fundamental circuits. Let \( C_r \) be the \((0-1)\)-fundamental circuit matrix with \( m \) columns and \( m-(n-1) \) rows where

\[
C_r(i,j) = \begin{cases} 
1 & \text{if } e_j \in \text{ circuit } i \\
0 & \text{otherwise.}
\end{cases}
\]

A cut-set is any subset \( U \) of edges whose deletion disconnects the graph. \( U \) is called minimal if \(|U|\) is minimum.

The \( n-1 \) minimal cut-sets with respect to \( T \) that contain one and only one edge of \( T \) are called fundamental cut-sets. Let \( T_f \) be the \((0-1)\)-fundamental cut-set matrix with \( m \) columns and \( n-1 \) rows where

\[
T_f(i,j) = \begin{cases} 
1 & \text{if } e_j \in \text{ cut-set } i \\
0 & \text{otherwise.}
\end{cases}
\]

Now we can consider the matrix \( Y \) as \( Y=(Y_1|Y_2) \) where the columns of \( Y_1 \) are put as the chords of a fixed tree.