A NEW APPROACH TO THE MODELLING UNCERTAINTY PROBLEM
OF SYSTEMS DESCRIBED IN STATE SPACE FORM

David Bensoussan
Ecole de technologie supérieure
Université du Québec
4750, rue Henri-Julien
Case postale 1000, Succursale E
Montréal, Québec, CANADA
H2T 1R0

ABSTRACT

Modelling of systems is generally done by frequency response methods or state variable methods. It is our object to show how frequency domain robustness results can be extrapolated to their state space counterpart. Using properties of input-output relations of systems and different compatible norms we will show how a corresponding frequency response robustness result can be applied. The method can be used to solve a certain class of non linear equations. It can apply to the control of non linear multivariable systems in order to better stability, sensitivity as well as decentralized control results. It can also apply to assess the state feedback, the output feedback and the observer with regard to the robustness problem.

1. INTRODUCTION

Multivariable control theory evolved in the sixties, using the state variable approach. This approach together with growing computer technology gave rise to tremendous research. Interesting results on system stability, controlability, observability, reachability and detectability were developed. This was a sharp contrast to the single input-single output frequency response approach involving polynomial approaches, Nyquist criterion, and root locus methods.

However, many of the answers given by state space methods lack the suppleness of multivariable methods as they apply to well defined models with no modelling uncertainty. Adaptive control is a partial response for the modelling uncertainty problem as far as parametric uncertainty is concerned. Clearly, in any state space representation (A, B, C, D), there is no way to predict the behaviour of eigenvalues whenever the matrix representation is modified to (A+ΔA, B, C, D). On the other hand, frequency response methods apply better to the uncertainty problem: in the case of a single input single output Nyquist diagram for instance, a Nyquist plot could be replaced by some Nyquist band representing the modelling uncertainty at each frequency.
Multivariable frequency response methods such as the inverse Nyquist area [1], the multivariable Nyquist criterion [2], and the multivariable root locus [3] are concerned mainly with system stability. However, the input output approach to systems [4,5,6,7,8] which apply to any normed algebraic representation of systems fit particularly to the frequency response setting. Such an approach allows us to handle the problem of modelling uncertainty. It is our purpose to show how multivariable frequency response uncertainty methods can be extrapolated to the multivariable state space uncertain models case.

2. Mathematical notations

We shall consider frequency responses defined in Hardy spaces, namely space $H^0_{\mathbf{u}}$, $H^0_{\mathbf{u}}$, and $H^0_{\mathbf{u}}$. $H^0_{\mathbf{u}}$ is the space of $n \times n$ matrices of frequency responses in $H^0_{\mathbf{u}}$, i.e. frequency responses which are holomorphic and bounded in some right half plane $\text{Re}(s) > 0$, $\alpha > 0$. $H^0_{\mathbf{u}}$ is the space of $n \times n$ matrices of elements in $H^0_{\mathbf{u}}$, i.e. frequency responses which are holomorphic and bounded in the open right half plane $\text{Re}(s) > 0$. $H^0_{\mathbf{u}}$ is the space of $n$-tuple vectors whose elements $u_1, u_2, \ldots, u_n$ are in $H^0_{\mathbf{u}}$, i.e. frequency responses which are holomorphic and bounded in $\text{Re}(s) > 0$. Frequency responses in $H^0_{\mathbf{u}}$ will be normed as follows:

$$||u_2||_2 = \frac{1}{\omega} \int_0^\infty ||u_1(\omega)||^2 \, d\omega$$

$$||u_1||_2 = \int_0^\infty ||u_1(\omega)||^2 \, d\omega \quad i = 1, 2, \ldots, n$$

We underline the fact that the $H^0_{\mathbf{u}}$ norm is equivalent to the $L^2$ norm, i.e.

$$||u_1||_2 = \int_0^\infty ||u_1(t)||^2 \, dt$$

Functions $T$ in $H^0_{\mathbf{u}}$ are normed as follows

$$||T||_\infty = \sup_{u \neq 0} \frac{||Tu||_2}{||u||_2}$$

$$||T||_\infty = \sup_{j} \sigma(T(j\omega))$$

where $\sigma(.)$ represents the maximal singular value of the matrix $T(j\omega)$.

We introduce the matrix $G(T)$ whose elements are $||T_{ij}||_\infty$ and a new norm $g(T) = \sigma\{G(T)\}$.

It has been shown [9] that such a norm is compatible with the $H^0_{\mathbf{u}}$ norm, i.e.

$$||T||_\infty \leq g(T) \leq n ||T||_\infty$$