Incremental Models of Updating Data Bases

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ABSTRACT

This paper introduces a generalization of weak model-theoretic forcing of [RobT1] and [Kei73]. This generalized forcing preserves classic properties of weak model-theoretic forcing, e.g. Generic Set Theorem (Thm. 3.23), Generic Model Theorem (Thm. 3.24, Thm. 4.8), and Henrard's Theorem (Thm. 5.8). It is applied in this paper to investigate a deductive model for updating a deductive data base with incomplete information, whose possible variations are restricted to certain finite sets of atomic or negated atomic first-order sentences. Moreover, the paper introduces the notion of pragmatic truth pertinent to those models, and characterizes it in terms of generalized forcing (Thm. 5.11).

In conclusion, the paper offers (Thm. 8.4) two semantic and two syntactic characterizations of the V-fragment of minimal entailment.

1 Introduction

Non-standard treatment of negation has a short but tempestuous history in Artificial Intelligence. A wide spectrum of solutions, from negation as failure to closed world assumption to circumscription to non-monotonic logics, has been proposed as a remedy for lack of complete information in the process of machine reasoning. Up to now, none of them seems fully satisfactory.

In this paper we present a different approach which is based on a syntactic construction of prioritized, conservative expansion. Inspired by ideas of Henrard [Hen73], we employ conservative expansions to introduce a family of weak generalized model-theoretic forcings. These forcings show promise as useful tools for formulation and analysis of reasoning paradigms pertinent to non-standard negation.

In order to describe their possible applications to the theory of update in deductive data bases, we consider a forcing-based incremental model of a data base introduced in [Suc84]. In this model the data base consists of a fixed part Σ, represented by a set of universally quantified sentences in a first-order language L, and of a variable part p (called increment in this paper), represented by a set of atomic and/or negated atomic sentences of L. The range of variableness of p, fixed for every particular model and
for every \( \Sigma \), defines the class of updates which may virtually happen in the lifetime of the data base. The fact that not all finite sets of atomic and negated atomic sentences are allowed as increments motivates us to introducing the notion of \textit{pragmatic truth}.

Pragmatic truth is the key concept in our model of updating data base. It appears, at least within this model, to be more adequate and useful than the classic notion of truth. Therefore, we construct a family of non-monotonic logics for reasoning from data bases of the form \( \Sigma \cup p \), complete with respect to pragmatic truth within the scope of universal sentences. As the reader will see, \textit{not} in these logics means \textit{it cannot happen that}.

In the last part of the paper we focus on two particular cases of such general models. Firstly, we allow as increments finite sets of atomic or negated atomic sentences of \( L \). The weak model-theoretic forcing of [Rob71], a special instance of our general one, forms an adequate basis for this case. This case was a subject of initial study in [Suc84] and [Suc89c]. Secondly, we restrict increments to negative sentences, obtaining a proper generalization of the closed world data bases of [Rei78] and [Min82]. In this second case, negative forcing, a construction formally beyond the general scheme of [Kei73], proves its usefulness. In particular, we demonstrate a direct relationship between pragmatic truth, closed world assumption, and another well known source of non-monotonism and non-standard behavior of negation: the minimal model semantics. These results, which we prove by applying our negative forcing technique, are refinements of those known in the literature, e.g. [She88].

The paper is organized as follows. Sections 2 and 3 provide technical results used in the rest of the paper. They introduce concepts of generalized forcing and pragmatic truth, and investigate their abstract properties. Section 4 concerns a special case of generalized forcing, whose instances are used in the rest of the paper. Sections 5, 6, and 7 focus on models of updating deductive data base. The role of pragmatic truth in these models and its relationships with generalized forcing are exposed. A strong and consistent version \textit{cw}\textsubscript{as} of the closed world assumption is characterized there. Section 8 investigates properties of minimal model semantics. Its main result (Thm. 8.4) states that for \( \forall \)-sentences, minimal entailment is equivalent to positive pragmatic entailment, to negative forcibility, and to entailment under \textit{cw}\textsubscript{as}.

## 2 Prioritized Conservative Expansions

We consider a first-order countable language \( L \). We usually follow the notations of [Bar78], Chaps. A1, A2, A4, and B1. Because we have to use logical connectives and quantifiers both in \( L \) and in the language of this article, outside \( L \) we occasionally use \( \land, \lor, \) and \( \sim \) in the sense of bounded quantifiers (\textit{for all} and \textit{there exists}, respectively) and negation. E.g.,