Topological properties of observability for a system of parabolic type

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ABSTRACT

The purpose of the present paper is to demonstrate topological properties of observable regions in a distributed parameter system. A parabolic partial differential equation with constant coefficients is considered. According to Sakawa's definition, observability is defined to be the possibility of the unique determination of the initial value by point measurements, or by spatially averaged measurements. Furthermore, n-mode observability is defined to be the possibility of the unique determination of the coefficients corresponding to the first n eigenvalues, based on the expansion of the solution by eigenfunctions. Then it is proved that n-mode observability is generic, that is, open and dense, whereas observability is shown to be dense in the whole space of measurements. In case of point measurements, it is shown that observability is valid almost everywhere with respect to the Lebesque measure. Moreover genericity of n-mode controllability and the related properties of controllability will be shown for the dual systems with controls.

1. Introduction

The problem of observability in distributed parameter systems has a different aspect from that in lumped parameter systems, because the former includes the specification of the spatial distribution of measurements, which we need not take into account for ordinary differential equations. For example, in distributed systems we have some local information of the state variable such as the point measurement which should be extended to the whole spatial domain. Therefore some

efforts have been devoted to the unique determination of the state from local measurements.

Goodson and Klein [1] considered the problem of uniqueness with respect to point observation. Moreover they proposed the definition of n-mode observability, which means the coefficients that correspond to first n eigenvalues in the eigenfunction expansion of the initial state is uniquely determined. Furthermore, Sakawa [4] considered a broader class of parabolic systems and gave the conditions of observability with respect to point measurement and spatially averaged measurement.

In view of their results, the measurement space can be divided into two regions, one where observability holds and the other where some portions of the state is "unobservable". Here a problem of topological properties of the observable region arises. For example, in case of lumped parameter systems, observability has been proved to be generic, that is, open and dense in the whole domain of definition (cf. Wonham [6]).

We consider here this problem with respect to a class of parabolic differential equations and examine whether observability and n-mode observability are generic, dense, or not in the space of measurement.

2. Preliminary consideration

This section depends mainly on Sakawa [4]. Let D be an open bounded region in n-dimensional Euclidean space $\mathbf{R}^n$ ($n > 0$) with a smooth boundary ∂D. Then we consider the following system:

$$\frac{\partial u}{\partial t}(t,x) = Au(t,x) \qquad (t,x)\in(0,T)\times D \tag{1}$$

$$A = \Delta - a_0 = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i} - a_0 \tag{2}$$

$$c_1 u(t,\xi) + (1-c_1)\frac{\partial u}{\partial \nu}(t,\xi) = 0 \qquad (t,\xi)\in(0,T)\times\partial D \tag{3}$$

$$0 \le c_1 \le 1$$

where a_0 is a real constant or an analytic function, c_1 is a real constant, and ν is the exterior normal to the boundary ∂D.

We assume the initial condition to be

$$u(0,x) = u_0(x) \qquad x \in D \tag{4}$$