ELLIPSOIDAL APPROXIMATIONS IN PROBLEMS OF CONTROL

István Vályi
International Institute for Applied Systems Analysis
A-2361 Laxenburg
Austria

Abstract

The subject of study in this paper is an adaptive control problem involving uncertainties. It is a special case of the one considered in the paper by Kurzhanski [1], in the present volume. The system is described by differential inclusions and, accordingly, its solution, a feedback control ensuring that certain feasibility constraints be fulfilled, is sought in the form of a set valued map. We apply recent results of ellipsoidal calculus to develop an easily implementable algorithm that gives approximations to the known exact formulae. The paper is therefore an attempt to carry out the program proposed in the above mentioned article.

1. Introduction

General convex sets are difficult to handle because their analytical description involves an infinite number of scalar parameters. In contrast to this, the family of ellipsoids can be identified by the coordinates of their center and a positive definite matrix representing their "shape". Ellipsoids are well suited for using as approximates of compact convex sets for the reason that many operations over convex sets can be followed in a relatively easy way by operations over their estimating ellipsoids. The idea was first used in the late sixties for estimating the propagation of numerical errors by Faddeev and Faddeeva [2] and in the study of uncertain dynamical systems by Schweppe [3]. After a decade without much activity in the field, new results have been obtained by Kurzhanski, Chernousko and others, an indication of renewed interest. Now, in addition to the known ellipsoidal approximations for the reachable sets of nonconstrained linear systems [4], [5], [6], [7], analogous results for both reachable sets and viable domains are available in the constrained case.

As indicated in the abstract, the solution of the problem that we shall consider is known, i. e. formulae are given for the computation of the support function of the control at each instant. The calculations involved are, however, very complex. (See also Kurzhanski and Nikonov [8]). Our aim here is to obtain an approximate solution in a simpler, and more constructive way. This is done through two steps. The first is to change to a surrogate problem in order to get rid of infinite operations involved in the original construction, and the second is to approximate the

solution of this problem with the intersection of a finite number of ellipsoids.

Accordingly, we consider the differential inclusion

$$\dot{p}(t)=C(t)p(t)+u(t) \qquad t\in\mathbf{T}=[t_0,t_1] \tag{1.1}$$

with the initial condition

$$p(t_0)\in\mathbf{P}^{(0)} \tag{1.2}$$

and the constraint on the controls of the form

$$u(t)\in\mathbf{V}(t) \qquad t\in\mathbf{T}\ .$$

Additionally, we require first that a viability condition of the form

$$p(t)+\mathbf{Q}[t]\subset\mathbf{K}(t) \qquad t\in\mathbf{T}^* \tag{1.3}$$

is met, with $\mathbf{T}^*\subset\mathbf{T}$ being finite, i. e.

$$\mathbf{T}^* = \{\ \tau_i\in\mathbf{T} : i\in\overline{1,r}\ \}$$

and $\mathbf{Q}[t]\subset\mathbf{R}^n$, $t\in\mathbf{T}$ consisting of all the values $q(t)\in\mathbf{R}^n$ that are compatible with incoming measured information represented by the function

$$y : \mathbf{T} \ \rightarrow\ \mathbf{R}^m\ .$$

As information arrive in real time, at the instant $t\in\mathbf{T}$, only the function

$$y_t\ :\ [t_0,t]\cap\mathbf{T}\ \rightarrow\ \mathbf{R}^m$$

$$y_t(\tau)=y(\tau)$$

is available. The variable q is defined by:

$$\dot{q}(t)\in A(t)q(t)+\mathbf{P}(t) \qquad t\in\mathbf{T} \tag{1.4}$$

$$q(t_0)\in\mathbf{Q}^{(0)} \tag{1.5}$$

$$y(t)\in G(t)q(t)+\mathbf{R}(t) \qquad t\in\mathbf{T}. \tag{1.6}$$

The family of measurements $y(t)\in\mathbf{R}^m$, $t\in\mathbf{T}$ that are compatible with the system (1.4), (1.5) and (1.6) will be denoted by $\mathbf{Y}$.

Second, we also want that the trajectory arrives to a given set at the final instant:

$$p(t_1)\in\mathbf{M}\ . \tag{1.7}$$

We suppose that the mappings

$$C : \mathbf{T} \rightarrow \mathbf{R}^{n\times n}$$