Dynamic Packet Routing on Arrays with Bounded Buffers

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Abstract. We study the performance of packet routing on arrays (or meshes) with bounded buffers in the routing switches, assuming that new packets are continuously inserted at all the nodes. We give the first routing algorithm on this topology that is stable under an injection rate within a constant factor of the hardware bandwidth. Unlike previous results, our algorithm does not require the global synchronization of the insertion times or the retraction and reinsertion of excessively delayed messages and our analysis holds for a broad range of packet generation stochastic distributions. This result represents a new application of a general technique for the design and analysis of dynamic algorithms that we first presented in [Broder et al., FOCS 96, pp. 390-399].

1 Introduction

The rigorous analysis of the dynamic performance of routing algorithms is one of the most challenging current goals in the study of communication networks. So far, most theoretical work on this area has focused on static routing: A set of packets is injected into the system at time 0, and the routing algorithm is measured by the time it takes to deliver all these packets to their destinations, assuming that no new packets are injected into the system in the meantime (see Leighton [8] for an extensive survey). In practice however, networks are rarely used in this “batch” mode. Most real-life networks operate in a dynamic mode whereby new packets are continuously injected into the system. Each processor usually controls only the rate at which it injects its own packets and has only a limited knowledge of the global state.

This situation is better modeled by a stochastic paradigm whereby packets are continuously injected into the system according to some inter-arrival distribution, and the routing algorithm is evaluated according to its long term behavior.

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In particular, quantities of interest are the maximum arrival rate for which the system is stable (that is, the arrival rate that ensures that the expected number of packets waiting in queues does not grow with time), and the expected time a packet spends in the system in the steady state. The performance of a dynamic algorithm is a function of the inter-arrival distribution. The goal is to develop algorithms that perform close to optimal for any inter-arrival distribution.

Several recent articles have addressed the dynamic routing problem, in the context of packet routing on arrays [7, 6, 9], on the hypercube and the butterfly [12] and general networks [11]. The analysis in all these works assumes a Poisson injection rate and requires unbounded queues in the routing switches (though some works give a high probability bound on the size of the queue used [7, 6]). Unbounded queues allow the application of some tools from queuing theory (see [4, 5]) and help reduce the correlation between events in the system, thus simplifying the analysis at the cost of a less realistic model. Clearly bounded buffers in the routing switches is a setting that most accurately models real networks.

A general technique for the design and analysis of dynamic packet routing algorithms has been developed in [1]. The crux of that work is a general theorem showing that any communication scheme (a routing algorithm and a network) that satisfies a given set of conditions, defined only with respect to a finite history, is stable up to a certain inter-arrival rate. Thus, the analysis of the long term behavior of a dynamic algorithm is reduced to a simpler question of analyzing a finite execution of the algorithm. Furthermore, this technique also gives a bound on the expected routing time in the stable state. The theorem applies to any inter-arrival distribution: the stability results and the expected routing time of a packet inside the network depend only on the inter-arrival rate. The waiting time in queues depends on the inter-arrival distribution and an explicit relation is given in the main theorem of [1].

To apply the general technique one needs to present an algorithm whose performance analysis (on finite segments) satisfies the conditions of the theorem. Several applications of the general technique to routing algorithms for low diameter networks such as the butterfly have been demonstrated in [1]. Here we present the first application to arrays: we consider an $n \times n$ mesh of routing switches with bounded buffers. Each routing switch node is connected to a processor. A processor has its own queue for the packets generated by it that are waiting to enter the network. We can assume this processor queue to be unbounded. (De facto, this queue is finite and when it becomes full the processor stops generating new packets.) Other packets pass only through the routing switch. The routing switch has a routing queue stored in a bounded buffer where packets waiting to be routed are placed. For simplicity we assume that packets have random destinations. This assumption can be relaxed as long as no destination is overloaded with packets.

**Theorem 1.** There is a packet routing algorithm for the $n \times n$ mesh with bounded buffers that is stable for any inter-arrival distribution with expectation at least $Cn$ for some fixed constant $C$. The expected time a packet spends in the network