Acquisition of Useful Lemma-Knowledge in Automated Reasoning

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Abstract. This paper presents a method for solving “hard” problems with automated theorem provers. Main principle is the support of a conventional brute-force search by lemma-knowledge, which is generated and elicited by the prover system. The performance of the proposed method depends critically on the usefulness of the elicited lemmata for the actual proof task. In this context an evaluation function called information measure is introduced, which relates the effort required for the production of a lemma $f$ to the problem relevancy of $f$. Experiments show its high potential.

1 Introduction

The last years have seen significant improvements in automated theorem proving. Present-day systems are sometimes already a serious competition to humans [12]. But despite of their increased performance and some spectacular successes humans seem to be generally still superior to computers. State-of-the-art theorem provers (TPs) are totally lost in the case of problems requiring “complex” proofs, though they are able to find existing “simple” solutions frequently faster than humans. This different behaviour of man and machine can be traced back to different paradigms. The human ability to perform a highly selective search is well suited to tackle “hard” problems; on the contrary, TPs tend to make a brute-force search and therefore often become an easy victim of the overwhelmingly large search space (combinatorial explosion).

In order to make TPs more powerful, it is suggestive to reduce the size of the search space which has to be processed for obtaining a solution. The most effective method in this respect seems to be the establishment of a control instance, which guides the search by avoiding the exploration of apparently uninteresting parts of the search space. All decisions of such a control instance are based on some kind of knowledge. Both high-level knowledge represented as plans [2, 14] and low-level knowledge encoded in neural nets [6, 7] are already under intense study. In this paper the promising usage of medium-level knowledge will be explored. The key idea is to replace some traditionally inferred parts of the original proof by problem-specific lemmata, which represent combinations of basic inference steps as newly introduced single basic inference steps. The resulting
"simplified" proofs will offer the chance to solve more problems within a reasonable time than before. This approach resembles the kind of reasoning performed by human experts, which can make larger steps compared to beginners [3]. One can hope that the support of a brute-force search by lemma-knowledge combines individual advantages of the search-intensive and the knowledge-intensive extremes.

Unfortunately, after adding a simple brute-force generated lemma set to the original axiomatization, the theorem prover usually requires more time for constructing a solution than before [9]. This phenomenon is explained by the introduced redundancy. Constraints applied in the original proof, for example, could be useless in the new situation, if their application needs insight into the proof parts substituted with lemmata. We must avoid such an uncritical use of numerous lemmata, since otherwise the disadvantages of search space enlargement will be overwhelming. But a strict limitation of the number of added lemmata seems useless as long as reliable algorithms for selecting lemmata with high value for the construction of the final proof are missing. Of course it is possible to support such a lemma selection procedure interactively by human experience. Generalized to a complete guidance of the search this method is practiced, for example, in Isabelle [13] or in KIV [15]. It works surprisingly good in practice, but it is unacceptable in many applications. The work load on the human expert is remarkable, because he must be informed about the progress of the search and supply the main ideas for the proof. Thus a completely automated lemma selection procedure would be of great help. Such a procedure is described in this paper. For the sake of simplicity we will restrict on the special case of unit-lemmata, which are lemmata consisting of only one literal. The selection procedure is based on a lemma evaluation in accordance with the so called information measure newly introduced here. Avoiding both trivial and irrelevant lemmata, this approach combines the strength of traditional automated TPs with human-like capabilities. First experiments yield encouraging results.

The paper is structured as follows. Section 2 is devoted to the main ideas of lemma selection. In Sect. 3, an implementation of the proposed lemma selection procedure is presented. Section 4 contains an evaluation of this implementation. The paper ends with a brief outlook in Sect. 5. Because analytical predictions are only feasible in simple toy-models, the paper focuses on the basic ideas and the application of the lemma selection rather than on formal background and theoretical considerations.

2 Principles of Lemma Selection

The usage of lemmata should reduce the necessary effort for finding a proof. Consequently, a lemma should be considered as suitable if it enables a simplification of the proof, which can outweigh the simultaneous growth of search space due to the introduced redundancy. If many elements of the lemma set added to the original axiomatization are suitable in this sense, then the proof can be found probably (much) faster [5]. But such a heuristic limitation of the search space