SCALING THEORY OF THE ORGANIZED PHASE OF SPIN GLASSES

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ABSTRACT

The ordered phase of short-range spin glasses is described in terms of the scaling behaviour associated with a zero-temperature fixed point. The main ingredient of the theory is the exponent $\gamma$ which describes the growth with length scale $L$ of the characteristic coupling at zero temperature, $J(L) \sim L^{\gamma}$. The exponent $\gamma$ is estimated numerically for dimensions $d=2,3$. For Ising spin glasses we find $\gamma \approx -0.3$ for $d=2$ and $\gamma \approx 0.2$ for $d=3$, implying scaling to weak (strong) coupling for $d=2(3)$, i.e. the "lower critical dimension" $d_\perp$ satisfies $2 < d_\perp < 3$. For $d < d_\perp$, $\gamma$ determines the divergence of the correlation length for $T=0$, while for $d > d_\perp$, it determines the large scale properties of the ordered phase, such as the long-distance behaviour of connected correlation functions, $G(r) \sim r^{-\gamma}$, and the singular response to a weak magnetic field, $m_{\text{sing}} \sim h^{d/(d-2\gamma)}$. The decay of the connected correlation functions at large distances implies that the pure-state overlap distribution function $P(q)$ is trivial, in contrast to the Sherrington-Kirkpatrick model. The dynamics of the system are also discussed, as is the extension to vector spin models.
I. INTRODUCTION

The mean-field theory of spin glasses, as realised in the Sherrington-Kirkpatrick (SK) model\(^1\), is now fairly well understood. The solution proposed by Parisi\(^2,3\) yields a rich structure in the ordered phase, described by an infinite number of pure states related by an ultrametric topology\(^4\).

Much less is known about the properties of spin glasses with short-ranged interactions. Indeed, until recently no consensus existed as to whether these systems exhibit a phase transition at all in three dimensions. A number of recent numerical studies, including phenomenological scaling\(^5,6\) and Monte Carlo simulations\(^7,8\), appear to have at last resolved this issue: the three-dimensional Ising spin glass exhibits a phase transition at a non-zero temperature, while in two dimensions a phase transition occurs at T=0 only - the "lower critical dimension" \(d_c\) satisfies \(2<d_c<3\). For short-range vector spin models\(^9\), it seems that \(d_c>3\).

In this article a theory of spin glasses is presented, based on a "one-parameter-scaling" picture, the central concept being that of a scale-dependent coupling \(J(L)\). Since the interactions in spin glasses are random variables, we introduce a distribution function \(P_L(J)\), whose scale width is \(J(L)\), for the couplings at length scale \(L\). The equilibrium properties in the ordered phase are determined from an exponent \(\gamma\) which describes the scale dependence of \(J(L)\) at zero temperature, \(J(L) \sim J\text{L}^\gamma\). The sign of \(\gamma\) determines whether the system scales to weak or strong coupling for \(L \rightarrow \infty\), i.e. whether \(d>d_g\) (\(\gamma>0\)) or \(d<d_g\) (\(\gamma<0\)). For \(d>d_g\), \(\gamma\) determines the divergence of the correlation length for \(T \rightarrow 0\): \(\xi = T^{-\nu}\), \(\nu = -1/\gamma\). For \(d<d_g\), the system flows to the strong coupling (zero temperature) fixed point for all \(T<T_c\), and the exponent \(\gamma\) determines the decay of correlations in the ordered phase. \([<(S_0 S_T)^2> - <S_0>S_T>^2]\text{av} = r^{-\gamma}, r \rightarrow \infty\), and the dependence of the non-analytic part of the magnetisation on the magnetic field, \(m_{\text{sing}} = h^d/(d-2\gamma), h \rightarrow 0\). The former result was first derived by Fisher and Huse\(^12\) using a droplet model, the latter by the present authors\(^13\) from a Renormalisation Group (RG) treatment in which the coupling \(J(L)\) is regarded as a coupling constant for "block spins" of linear dimension \(L\).

In the present article we recover and extend the Fisher-Huse results using RG terminology throughout for consistency. We note that the decay of the connected correlation functions at large distances implies that, in contrast to the SK model, the "pure state overlap distribution function"\(^3\) is trivial (as suggested previously from independent arguments\(^15\)), \(P(q) = \varphi(\delta(q-q(T)) + \delta(q+q(T)))\), where \(q(T) = [<(S_i)^2>]\text{av}\) is the Edwards-Anderson order parameter\(^14\). For small \(\gamma\), however, finite size corrections to this form can be large\(^15\). We also discuss the dynamics\(^12,46\) within the same spirit.

The absence of replica symmetry breaking in the ordered phase implies that there can be no Almeida-Thouless instability\(^16\) in low dimensions, i.e. a magnetic field destroys the spin-glass transition. We argue that this result must break down in high dimension, probably \(d=6\), since the AT line can be explicitly calculated.