The One-Dimensional Three-Body Problem: Numerical Simulations

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Abstract. The ergodic properties of a one-dimensional gravitational system belonging to the microcanonical ensemble are studied. This system, constituted of equal-mass particles, exhibits very strong binary structures which prevent the system to be ergodic and then to reach the theoretical curves predicted by Rybicki. The presence of the binary structure (called molecule) is examined through two criterions. The first one is given by a topological property based on the order relation of the one-dimensional systems; the second one is the internal energy of the molecule. At last the study of the molecule stability indicates that it strongly depends on initial conditions.

1. Introduction

One-dimensional systems have been studied since many years. Although unrealistic, they give some interesting ideas on the basic physical concepts. Such is the case for the study of ergodicity for the following reasons:

- The theoretical distribution functions of one-dimension systems have been determined for numerous systems; plasma (Lenard 1961), Boltzmann-gas (Rouet et al. 1993), gravitational systems (Rybicki 1971).
- In the case of some one-dimensional systems it is possible to perform an exact numerical code (Feix 1969), a property which is crucial to test the validity of the ergodic concept.

At last the very-long range field of one-dimensional gravitational particles keeps the system self-trapped and amplifies the formation and live of coherent structures such as binary structures observed in this study.

The validity of the concept of ergodicity has been numerically tested on a very simple system, that is, the Boltzmann gas. Such a system is constituted of point particles, of different masses, moving on a line and experiencing hard-core collisions (for equal masses, the result is trivial). The numerical results fit the theoretical one obtained,
by supposing the surface of equi-energy uniformly covered (this surface, or hypersurface, is drawn considering the "right" variable \( P\sqrt{m} \)), because the system has an order relation given by the position of the particles (in this case the particles cannot cross each other and the initial order position relation is preserved during the simulation).

These considerations give two interesting results: we obtain the equi-repartition of the energy whatever the number of particles is, and the position distribution function is independent of the particle masses (cf. Rouet et al. 1993 for more details).

Note that, in this case, for special ratio of particle masses we do not have a filling of the surface of constant total energy because the initial values of the particle velocities are preserved. We verify that the numerical results recover this property which indicates that the round-off errors are not too important, and, at least for this case, do not destroy this conservative set of values.

2. The One-Dimensional Gravitational Three-Body Problem

2.1 Model and Preliminary Results

In the case of a one-dimensional system, the particles are infinite plane sheets of uniform superficial density of mass \( \mu \), moving in the perpendicular direction of the sheet. As the field has no divergence at the origin, the particles are allowed to pass freely through each other.

This present study is limited to the case of \( N = 3 \) particles of the same mass \( \mu \).

This system has three invariants which are, the total energy, the total momentum (which can, without any loss of generality, be taken equal to zero), and then the position of the center of mass. Taking into account these integrals of motion, Rybicki derived the position and velocity distribution functions for microcanonical ensemble systems of \( N \) particles.

Numerical and theoretical results are compared in Figs. 1 for the velocity and position distribution functions. For this simulation (simulation A) the velocities of all the particles are initially equal to zero and the positions are respectively \( x_1 = -15, x_2 = 6.5 \) and \( x_3 = 8.5 \) in arbitrary units. The fitting is not good at all and the position distribution function looks like the superposition of two curves of systems containing each two particles.

Other simulations with different initial conditions have been performed but neither give a good agreement with the Rybicki curves.

Consequently it may exists, in addition with the three invariants, another constraint which prevents the system to be ergodic.