Theoretical and Numerical Investigation of the Stability of Flattened Galaxies

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Abstract. Quasi-linear theory is applied to the wave-star interaction of a differentially-rotating stellar disk of a galaxy. Under the influence of growing spiral waves the velocity dispersion of stars increases, and the resulting distortion in phase space leads to a decrease in the growth rate of the waves, and the Jeans instability ends. Due to interactions of stars with unstable waves the relaxation of the stellar disk of the Galaxy occurs in $\sim 10^9$ years. The theory is confirmed by $N$-body computer simulations.

In modern density wave theory, the spiral structure of galaxies results from a non-axisymmetric gravitational Jeans instability (see Lin & Bertin 1984 for review). Up to now, density wave theory has been developed only in the linear approximation. Weak nonlinear theory, i.e., nonlinear theory using a perturbation approach, can be used to explain a broad class of phenomena in the stellar disk of a galaxy. This approach was initially developed for the physics of an inhomogeneous plasma in a magnetic field (Krall & Trivelpiece 1973). For instance, under the action of growing waves, the average properties of a stellar disk may change; the “temperature” of the system which is measured by the kinetic energy of random motion may increase. This increase in turn leads to a stabilization of the gravitational instability as computer experiments have shown (Hohl 1972; Miller 1976).

To describe the properties of stellar disks in the nonlinear regime it is useful to consider the quasi-linear wave-star interaction that is not associated with the resonance condition $\omega = kv$, where $\omega$ is the wave frequency, $k$ is the wavenumber, and $v$ is the velocity.

Let us consider an inhomogeneous differentially-rotating stellar disk of a galaxy, taking a kinetic description as a basis. To describe the problem, it is useful to employ the well-developed mathematical formalisms in plasma physics (e.g., quasi-linear theory) which have dealt with similar problems. For a discussion on the formal analogy between the oscillations of a differentially-rotating stellar disk and the oscillations of a magnetic plasma see the mono-
graph by Fridman & Polyachenko (1984). By using the concepts of quasi-linear theory we can show that through the influence of growing spiral waves the stars will tend to diffuse in velocity and coordinate space according to the relation:

$$\frac{\partial f_0}{\partial t} = \alpha \frac{\partial^2 f_0}{\partial v^2} - \beta \frac{\partial^2 f_0}{\partial r^2}$$  \hspace{1cm} (1)

Here we have presented the distribution of stars in the form \( f(t) = f_0(t) + f_1(t) \), where \( f_0(t) \) changes slowly in time and the small perturbation \( f_1(t) \) changes rapidly. See Krall & Trivelpiece (1973), Griv (1992), and Grivnev (1988) for a more detailed derivation of Eq. (1). This equation has the following solutions for the non-resonant (or adiabatic) interaction of waves and stars:

$$f_0(v) \sim \frac{1}{\sqrt{\sigma_0^2 + |\mathcal{E}|}} \exp \left[ \frac{-v^2}{2(\sigma_0^2 + |\mathcal{E}|)} \right]$$  \hspace{1cm} (2)

and

$$n(r) \sim \frac{1}{\sqrt{n_0^2 - |\xi|\mathcal{E}}} \exp \left[ \frac{-r^2}{2(n_0^2 - |\xi|\mathcal{E})} \right]$$  \hspace{1cm} (3)

In these equations, \( n(r) \) is the surface mass density, \( \sigma_0 \) (\( n_0 \)) is the initial velocity (surface density) dispersion, and \( \mathcal{E} \) is the square of the wave amplitude.

According to Eqs. (2-3), the main body of the distribution is effectively heated and the surface density is redistributed by the unstable waves. The velocity dispersion of the non-resonant part of the distribution function increases and \( f_0(v) \) becomes less peaked, and the surface density becomes more peaked as the wave energy increases. The diffusion of the stars takes place because the stars gain additional oscillatory energy in the gravitational field of the unstable density waves.

According to Eqs. (2-3), under the action of growing waves the velocity dispersion of stars will increase and the density of the disk falls off exponentially. This distortion in phase space caused by the spiral waves leads to a decrease in the growth rate of the waves and eventually to the cessation of the instability. Therefore, from the theoretical point of view, the spiral waves in computer-generated galaxies have to be short-lived, and should dissipate after a few rotations of the disk system.

Our theoretical results have been verified by three-dimensional N-body simulations. We investigated the evolution of a model for an isolated thin disk of a galaxy by direct integration over a time span of equation of motion of identical stars. The cutoff radius \( r_c \) of the Newtonian potential was introduced in order to eliminate close encounters between the model particles. This “softening” parameter reduces the interaction at short ranges and puts a lower limit on the “size” of the particles.

At the start of the N-body integration a mass density variation given by