By a graph $G$, is meant a set of $n$-points, called the set of vertices, $V(G)$; and a set $E(G)$, of lines, or edges, joining some pairs of vertices, so that no pair of vertices is joined by more than one edge, and no edge joins a vertex to itself. When a pair of vertices in a graph are joined by an edge they are called adjacent. The adjacency matrix of a graph $G$, denoted $A(G)$, is a square 0 - 1 matrix of order $n$ whose rows and columns correspond to the vertices of $G$ and for which $A_{ij} = 1$ if and only if vertex $i$ and vertex $j$ are adjacent. Thus, for each of the graphs considered, the associated adjacency matrix is symmetric with zeros on the diagonal. The eigenvalues of a graph are the eigenvalues of its adjacency matrix, and hence are real. For any graph $G$, the eigenvalues will be denoted $\lambda_1(G) \geq \lambda_2(G) \geq ...$ in descending order, and $\lambda_1(G) \leq \lambda_2(G) \leq ...$ in ascending order. The complement, $\overline{G}$, of the graph $G$ is the graph described by $V(\overline{G}) = V(G)$, where two vertices in $\overline{G}$ are adjacent if and only if these two vertices are not adjacent in $G$. The valence of a vertex $v$ in a graph $G$ is the number of edges for which that vertex is an end-point. Two graphs $G$ and $H$ are said to be "L away from each other" if there exists graphs $\overline{G}$, and $\overline{H}$ such that $A(G) + A(\overline{G}) = A(H) + A(\overline{H})$ and every vertex of $\overline{G}$ and $\overline{H}$ has valence at most $L$. A sub-graph $G'$ of $G$ is the graph on the non-empty subset $V(G')$ of vertices of $V(G)$ where two vertices in $G'$ are adjacent if and only if they were adjacent in the original graph $G$.

The following notation will be used:

$\begin{array}{c}
\bullet \\
\end{array}$ will be a complete graph on $\ell$ vertices, or clique, abbreviated $K_\ell$, where every vertex is adjacent to every other vertex.
will be a graph formed by \( K_t \) and one more vertex adjacent to all the vertices of \( K_t \); that is, \( K_{t+1} \).

\( \tilde{K}_t \) will be the independent set of \( t \) vertices, abbreviated \( \tilde{K}_t \), in which no two vertices are adjacent.

\( \tilde{K}_2t \) will be a graph formed by two cliques on \( t \) vertices, where every vertex in each clique is adjacent to all other vertices, that is \( K_{2t} \).

\( \tilde{K}_t \) will be a graph formed by \( \tilde{K}_t \) and one more vertex adjacent to all the vertices of \( \tilde{K}_t \).

In short, a solid line joining graphs \( A \) and \( B \) forms a graph where every vertex in \( V(A) \) is adjacent to every vertex in \( V(B) \).

A. J. Hoffman proved the following

**Theorem:** Let \( Q \) be an infinite set of graphs, then the following statements about \( Q \) are equivalent:

1. There exists \( \lambda \) such that \( \lambda(G) \geq \lambda \), \( \forall G \in Q \). Where for any \( G \), \( \lambda(G) \) is the least eigenvalue of \( A(G) \).
2. There exists a positive integer \( t \) such that no \( G \in Q \) contains either \( \tilde{K}_t \), or \( \tilde{K}_2t \), as a sub-graph.
3. There exists a positive integer \( L \), such that for each \( G \in Q \) there exist graphs \( G \) and \( H \) with the following being true:
   3a) \( A(G) + A(C) = A(H) \).
   3b) Every vertex of \( C \) has valence at most \( L \); and \( H \) contains a family of cliques \( K_1, K_2, \ldots \) such that:
   3c) Each edge of \( H \) is in at least one \( K_i \),
   3d) Each vertex of \( H \) is in at most \( L \) of the cliques \( K_1, K_2, \ldots \)
   3e) \( |V(K_i) \cap V(K_j)| \leq L \), \( i \neq j \).

The main result of the present investigation is the following

**Theorem 1:** Let \( Q \) be an infinite set of graphs, then the following statements about \( Q \) are equivalent:

1. There exists a real number \( \lambda \) such that \( \lambda_2(G) \leq \lambda \) for every \( G \in Q \).
2. There exists a positive integer \( t \) such that for each \( G \in Q \) the