1. Introduction. Our object here is to examine the convolution device introduced in [1] and to discuss several applications it has found. We shall use throughout the standard notations of harmonic analysis as found in [2]. In particular when G is a locally compact abelian group \( A(G) \) and \( B(G) \) denote the Banach algebras of absolutely convergent Fourier series on \( G \) and Fourier-Stieltjes transforms on \( G \) respectively.

Let \( B \) be a Banach algebra realised as an algebra of functions on its maximal ideal space \( X \). Let \( Y \) be a closed subspace of \( X \) and let \( B|_Y \) denote the Banach algebra of restrictions with the quotient norm. In the case where \( B|_Y \) is isomorphic to \( C_0(Y) \) we say that \( Y \) is a set of interpolation for \( B \) and we are assured the existence of a constant \( a (0 < a < 1) \) such that

\[
a \|f\|_{B|_Y} \leq \|f\|_\infty \leq \|f\|_{B|_Y} \quad \forall f \in C_0(Y),
\]

by virtue of the closed graph theorem. A set \( Y \) satisfying this condition is called an \( I_\alpha \) set. Alternatively, by the Hahn-Banach theorem the condition has the dual formulation

\[
\|u\|_{(B|_Y)^*} \geq a \|u\|_M \quad \forall u \in M(Y).
\]

From this it may be seen that \( Y \) is an \( I_\alpha \) set if and only if all the totally disconnected subsets of \( Y \) are \( I_\alpha \) sets. If \( Y \) and \( Z \) are interpolation sets it is natural to ask whether \( Y \cup Z \) is. We may assume that \( Y \) and \( Z \) are disjoint and totally disconnected in the sense that it suffices to find \( a > 0 \) such that \( Y' \cup Z' \) is an \( I_\alpha \) set whenever \( Y' \) and \( Z' \) are disjoint totally disconnected subsets of \( Y \) and \( Z \) respectively. It is easy to see that \( Y \cup Z \) is an interpolation set if and only if there exists a separating function
$s \in \mathcal{B}$ such that

$$|s(y) - 1| \leq 1/3 \quad \forall y \in Y, \quad |s(z)| \leq 1/3 \quad \forall z \in Z.$$ 

In the case $Y \cap Z \neq \emptyset$ it will be necessary to construct $s$ for each $Y'$ and $Z'$ with uniform control of norm. In the case where $\mathcal{B}$ has no identity it suffices to find $s$ in the multiplier algebra of $\mathcal{B}$.

2. The convolution device. Let $K$ be a compact $\alpha$ set and let $\Omega$ be a set of continuous functions of unit modulus on $K$ which form a group under pointwise multiplication. For all $\delta > 0$ there exist functions $f_\omega (\omega \in \Omega)$ in $\mathcal{B}$ such that

1) $f_\omega (x) = \omega(x)$ $\quad \forall x \in K, \forall \omega \in \Omega.$

2) $||f_\omega||_B \leq a^{-1} (1+\delta) \quad \forall \omega \in \Omega.$

If we consider $f$ as a function on $X \times \Omega$ and denote $f_x(\omega) = f_\omega(x)$ the most we can say about $f_x$ is

3) $||f_x||_{C(\Omega)} \leq a^{-1} (1+\delta) \quad \forall x \in X,$

although for $x \in K$ we note that $f_x$ is an algebraic character on $\Omega$.

Restricting attention for the moment to the case where $\Omega$ is finite we consider the convolution

$$g(x_\omega) = \int f(x, \omega \lambda^{-1}) f(x, \lambda) \, d\eta(\lambda),$$

where $\eta$ is the normalized translation invariant measure on $\Omega$.

Clearly we have

1') $g_\omega (x) = \int (\omega \lambda^{-1})(x) \cdot \lambda(x) \, d\eta(\lambda) = \omega(x) \quad \forall x \in K, \forall \omega \in \Omega.$

2') $||g_\omega||_B \leq \sup_{\lambda \in \Omega} ||f_\lambda||_B^2 \leq a^{-2} \cdot (1+\delta)^2 \quad \forall \omega \in \Omega.$

For a fixed element $x \in X$ we have

3') $||g_x||_{A(\Omega)} = ||f_x \ast f_x||_{A(\Omega)} = ||f_x||_L^2(\Omega)$

$$\leq ||f_x||_\infty^2 \leq a^{-2} (1+\delta)^2.$$