1. Let \( \mu \) be a finite (non-zero) measure on a locally compact abelian group \( G \) with dual \( \Gamma = G^\wedge \), and suppose \( 0 \) is isolated in the range of the Fourier-Stieltjes transform \( \hat{\mu} \), or more precisely,

\[
(1.1) \quad 0 \text{ is isolated in } |0| \cup \hat{\mu}(\Gamma).
\]

Then what can one say about \( \mu \)?

Using Cohen's approach to the idempotent problem (as simplified by Amemiya and Ito) we shall show (1.1) implies \( \mu \) has a non-zero component of a very nice sort: there is a compact subgroup \( H \) of \( G \) for which \( \mu_H \), the part of \( \mu \) carried by the cosets of \( H \), is the convolve of a (non-zero) idempotent and an invertible. Thus in particular \( \hat{\mu}_H \) (whose range is related to that of \( \hat{\mu} \) in general, cf. [8]) is supported by an element of the coset ring of \( \Gamma \). As an application, suppose \( \mu \) is a measure on \( T^1 \) whose discrete part is not the convolve of a (non-zero) idempotent and an invertible; if \( \{ I_n \} \) is a sequence of disjoint intervals in \( \mathbb{Z} \) with lengths tending to \( \infty \) on each of which \( \hat{\mu} \) doesn't vanish identically, then for every \( \varepsilon > 0 \), \( |\hat{\mu}| \) assumes a value in \((0, \varepsilon)\) on all but finitely many \( I_n \).

The result arose from another unsuccessful attempt to answer the question raised in [7]; the measures \( \mu \) for which \( \mu * L_1 \) is closed, which satisfy (1.1), may well have the form indicated above for \( \mu_H \), but the measures satisfying (1.1) itself need not be of that form. For example, if \( E \) is any Sidon set, Drury's construc-

*Work supported in part by the National Science Foundation.
tion [5] yields a measure \( \nu \) with \( \hat{\nu} = 1 \) on \( E \) and \( |\hat{\nu}| < \frac{1}{2} \) off \( E \), so \( \delta_o - \nu \) satisfies (1.1), as does \( \mu \ast (\delta_o - \nu) \) for any \( \mu \) satisfying (1.1), while \( (\mu \ast (\delta_o - \nu))^\wedge \) vanishes exactly on \( E \cup \hat{\mu}^{-1}(0) \). Thus one can alter considerably the set on which the isolated value \( O \) is assumed (at least with \( G \) compact).

To state our result precisely, recall that for a closed subgroup \( H \) of \( G \), any \( \mu \in \mathcal{M}(G) \) can be uniquely decomposed (via exhaustion, as usual) as

\[
\mu = \mu_H + \mu',
\]

where \( \mu_H \) is carried by cosets mod \( H \) and \( \mu' \) vanishes on all Borel subsets of cosets mod \( H \) [10]. Our main result is then

**Theorem 1.1.** For any non-zero measure \( \mu \) on a l.c.a. group \( G \) satisfying (1.1) there is a compact subgroup \( H \) of \( G \) and characters \( \gamma_1, \ldots, \gamma_n \) in \( \Gamma \) for which

\[
\mu_H = \eta \ast \lambda, \quad \eta = \left( \sum_{1}^{n} \gamma_i \right) m_H, \quad \lambda \in \mathcal{M}(G)^{-1}
\]

(where \( m_H \) is the normalized Haar measure on \( H \) and \( \mathcal{M}(G)^{-1} \) denotes the invertible elements of \( \mathcal{M}(G) \)).

In what follows it will be convenient to use multiplicative notation for the group operation in \( \Gamma \), and to omit the usual conjugation in defining the Fourier-Stieltjes transform:

\( \hat{\mu}(\nu) = \int \nu \, d\mu = \mu(\nu) \). Finally \( G^d \) will denote the discrete version of \( G \).

2. We begin with a key lemma which is essentially proved (but not stated) in Cohen's work [2] and in the Amemiya-Ito paper [1], and which was no doubt known to those authors.

**Lemma 2.1.** Suppose \( X \) is a locally compact Hausdorff space,