Decission Procedure for Theories Categorical in $\aleph_0$

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The decision procedure for categorical theories suggested by well known test of R.L.Vaught [5] is not the simplest one. In many cases a simpler procedure may be obtained using a kind of elimination of quantifiers which is universal for theories categorical in $\aleph_0$. We start by study of models for theories categorical in $\aleph_0$. Then we pass to the decision procedure. At the end we make some suggestions concerning the interesting conjecture of decidability of every categorical theory having finite number of primitive notions.

1. Models categorical in $\aleph_0$

Categorical theories are given rather by looking at their models like dense ordering or atome - free boolean algebra. I shall mention some other examples. From the dense ordering we can easily pass to the betweenness relation $B(x, y, z)$ on graphs.
If a graph has a circle, then all points of the circle are equivalent with respect to tree places betweenness relation. Then we can introduce 4 places betweenness \( \text{Be}(x, y, z, v) \). If the graph has finite number of nodes, then its \( B \) (or \( \text{Be} \)) theory is categorical in \( \aleph_0 \) and decidable \([2]\). If the graph has infinite number of nodes, then its theory may be undecidable \([1]\).

Studying models we say that \( \mathcal{M} = \langle M, R \rangle \) is categorical iff the theory of \( \mathcal{M} \) (the set of true sentences in \( \mathcal{M} \)) is categorical. Hence (because of its completeness) a theory is categorical iff it is the theory of a categorical model.

Starting from the Ryll-Nardzewski \([4]\) main theorem one can characterize models categorical in \( \aleph_0 \) as uniform by decomposition in the following sense:

**Definition.** \( \mathcal{M} \) is \( n \) - parametrically uniform by decomposition if and only if for each \( n \) - tuple of parameters \( a_1, \ldots, a_n \in M \) there is a finite decomposition of \( M \):

\[
M = I_1 \cup \ldots \cup I_k
\]

such that every \( I_i \) \( (1 \leq i \leq k) \) is uniform in the logical sense with respect to the parameters \( a_1, \ldots, a_n \):

this means that for every two elements \( x, x' \in I_i \) and for