ABSTRACT. The feasibility of applying the restricted permutation representation method of Macdonald and Street to colouring the fundamental regions of 3-dimensional crystallographic groups is discussed. The tetragonal crystal class is used to illustrate a classification of colourings.

1. CONSTRUCTION OF SPACE GROUPS. Space groups have been constructed and classified in many ways using many different notations since the pioneering work of Bravais, 1850. Detailed classifications can be found in Buerger (2), Burckhardt (3), Hilton (4).

A space group is a 3-dimensional lattice into the cells of which symmetrical groups of objects such as molecules are introduced. The largest group of symmetries consistent with a given lattice and the objects in its cells is then a space group. There are 6 basic lattice types depending on the lengths and orientations of the three generators of minimum length of the lattice points. If the generating vectors are as shown in Figure 1, the 6 lattice types are given by the following conditions on $a$, $b$, $c$, $\alpha$, $\beta$, $\gamma$.

- **Triclinic**: $|a| \neq |b| \neq |c|$, $\alpha \neq \beta \neq \gamma$
- **Monoclinic**: $|a| \neq |b| \neq |c|$, $\alpha = \beta = \frac{\pi}{2}$
- **Orthorhombic**: $|a| \neq |b| \neq |c|$, $\alpha = \beta = \gamma = \frac{\pi}{2}$
- **Tetragonal**: $|a| = |b|$, $\alpha = \beta = \gamma = \frac{\pi}{2}$

![Diagram showing lattice types](image)
Hexagonal \[ |a| = |b| \quad \alpha = \beta = \frac{\pi}{2}, \; \gamma = \frac{\pi}{2} \]

Cubic \[ |a| = |b| = |c| \quad \alpha = \beta = \gamma = \frac{\pi}{2} \]

If a lattice cell generated as above contains no interior points it is called \textit{primitive}, \( P \), and the lattice points can be denoted by the set

\[ T = \{ a^\lambda b^\mu c^\nu | \lambda, \mu, \nu, \text{ integers} \} \]

In the 32 crystallographic groups of point symmetries the generating rotations are restricted to certain combinations of the angles \( 2\pi, \pi, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{\pi}{3} \). These 32 point groups can be combined with the appropriate lattices to form 230 three-dimensional space groups.

2. THE TETRAGONAL CRYSTAL CLASS. There are just 7 point groups which induce automorphisms of the tetragonal lattice, but not of lattices of lower symmetry.

\( 4, \; \bar{4}, \; \frac{4}{m}, \; \bar{4}m2, \; 4mm, \; 422 \) and \( \frac{4}{m} \frac{2}{m} \frac{2}{m} \).

In each case the symbols denote the generators of the group, 4 denotes a 4-fold rotation, 2 a 2-fold rotation, (with axis perpendicular to the 4-fold one) \( m \) in the numerator denotes a reflection in a plane through the axis of the 4-fold rotation, \( m \) in the denominator a reflection in a plane perpendicular to the axis of the 4-fold rotation. \( \bar{4} \) is generated by a roto-reflection, i.e., a 4-fold rotation combined with a reflection in a plane perpendicular to its axis. Each of these point groups is consistent both the primitive and the body-centred tetragonal lattice, giving the space groups \( \bar{P}4, \bar{P}4, \ldots \), \( \frac{4}{m} \frac{2}{m} \frac{2}{m} \), \( I4, \; \bar{I}4, \; \ldots \), \( \frac{4}{m} \frac{2}{m} \frac{2}{m} \). However this does not exhaust all the possibilities. There exist two further composite symmetry operations, screws and glides. A screw is a combination of a rotation and a translation parallel to the axis of rotation. If \( C \) is a 4-fold rotation with axis parallel to the lattice generator \( c \) and \( A \) is a 2-fold rotation with axis parallel to lattice generator \( a \) then the screws consistent with the tetragonal class are \( \frac{1}{4} C, \frac{1}{4} C, \frac{3}{4} C, \frac{1}{4} b \frac{1}{2} A \). These appear in space group names as \( 41, \; 42, \; 43, \; 21 \) respectively. A glide is a combination of a reflection and a translation parallel to the reflection plane and is indicated in the name of a space group by replacing the letter \( m \) by \( a, \; b, \; c \) or \( n \) depending on the direction of the translation part of the glide with respect to the lattice generators.