A better title for this paper would be: "An introduction to relational semantics for normal modal propositional logic". Some of the material goes back as far as the unpublished work of Lemmon and Scott [66]; much of it will be found in the work of Segerberg [68,71]. Nothing in the paper is original but it seems to me that a short introduction which (a) begins at the beginning and (b) concentrates only on normal modal propositional logics, might be useful. Readers who would like to know more of the genesis of the subject and the intuitive interpretation of the range of systems should consult Hughes and Cresswell [68]. Readers who would like a much more comprehensive presentation of the topics covered in the present paper may be referred to Segerberg [71].

2. Basic Syntax and Semantics

A language $\mathcal{L}$ for propositional modal logic consists of the following:

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1 The present paper covers the material in the first four of six lectures I gave at Monash in January 1974. The only other short paper I am aware of which can form an introduction to the techniques expounded in the present paper is Segerberg's [68]. Readers would be well advised to proceed to Segerberg [68] from the present paper and then tackle the more extended Segerberg [71]. An early paper presenting the completeness results of section 2 by a similar method is Makinson's [69].
1.1 A denumerable set \( P \) of proposition letters (sometimes called propositional variables). We refer to these as \( p, q, r, p_1, q_1, r_1 \ldots \) etc.

1.2 The five symbols, \( \land, \lor, \neg, \rightarrow, L \). These are called logical constants and must be distinct from the proposition letters.

The set \( S \) of sentences (or wff) of \( L \) is the smallest set satisfying

1.3 If \( p \) is a propositional letter then \( p \in S \)

1.4 If \( a \) and \( b \) are in \( S \) then so are \( \neg a, (a \lor b) \) and \( L a \).

We make use of the following abbreviations

1.5 \((a \supset b)\) for \((\neg a \lor b)\)

1.6 \((a \cdot b)\) for \((\neg a \lor \neg b)\)

1.7 \((a \equiv b)\) for \(((a \supset b) \lor (b \supset a))\)

1.8 \( Ma\) for \( \neg L \neg a\)

You will no doubt recognize the first three as one notation (that of Principia Mathematica) for ordinary propositional logic. You will also know that there are many others. In place of \( L \), \( \Box \) is frequently used. In place of \( M \), \( \Diamond \) is often used. The original intuitive meaning of \( L \) was 'it is necessary that' and of \( M \) 'it is possible that'.

All the logics which will be discussed in this paper will be stated in a language of the kind defined. Trivial variants are obtained by basing the language on other primitive symbols, but in the context of the matters we shall be discussing they are not essentially different.

By a normal modal logic (in \( L \)) is meant a set \( \Lambda \) of wff