Several constructions of Steiner triple systems (STS) with ovals are given. For every \( v \equiv 3 \) or \( 7 \) mod \( 12 \) there are STS's with hyperovals, for every \( v \equiv 1 \) or \( 3 \) mod \( 6 \) there are STS's with ovals, and for infinitely many \( v \equiv 1 \) or \( 3 \) mod \( 6 \) there are STS's without ovals. The ovals may be classified by their complementary sets, the so-called counterovals. Several questions remain open.

1. Introduction

Up to now arcs and ovals were mainly investigated in projective planes. In classical projective planes of odd order the famous theorem of B. Segre holds that each oval is a conic [13], [7]. Of course these concepts make sense in linear spaces resp. in partial linear spaces. A partial linear space is a finite incidence structure \((V, \mathcal{G})\) with point set \(V\) and line set \(\mathcal{G}\) with at most one line through any two points and at least two points on every line. It becomes a linear space if every unjoined point pair is considered as a new line. We write \(v\) for \(|V|\) and \(b\) for \(|\mathcal{G}|\).

Examples of partial linear spaces are the so-called group divisible designs (GDD), where the points are partitioned into classes such that two points are joined iff they are in distinct classes; in particular the transversal designs (TD) with \(k > 2\) classes such that every line intersects every point class. It is well known that then each class has exactly \(g\) points and there are \(g^2\) lines. Such a TD is called a TD\([k; g]\). The existence of a TD\([k; g]\) is equivalent to the existence of \(k - 2\) mutually orthogonal Latin squares. In the sequel we assume that in a partial linear space at least one line has more than two points.
Definitions: An arc in a partial linear space is a point set which intersects no line in more than two points. Obviously every arc is a subset of a maximal arc. For any point set B a line L is called a

\[
\begin{align*}
\text{subline} & \quad \text{of } B \text{ if } |L \cap B| = |L| \\
\text{secant} & \quad 2 \\
\text{tangent} & \quad 1 \\
\text{passant} & \quad 0.
\end{align*}
\]

An arc is called a hyperoval if it has no tangents, and an oval if there are tangents but at most one through any point of it. Let \( r_p \) be the number of all lines through a given point \( p \). If \( B \) is an arc and \( p \in B \), then the number of tangents of \( B \) through \( p \) is \( r_p - |B| + 1 \). If \( H \) is a hyperoval and \( x \in H \), then there are exactly \( |H|/2 \) secants through \( x \), and the number of tangents in a point \( p \in H \) is \( 0 = r_p - |H| + 1 \), i.e. \( r_p = |H| - 1 \).

An oval \( B \) in a linear space can be extended to a hyperoval only if each point \( x \in B \) is on exactly one tangent and all these tangents have a point in common.

If \( r_p \) is independent of \( p \) (e.g. in Steiner systems \( S(2,k;v) \) with exactly \( k \) points on every line, or in GDD's \( GD[k,g;v] \) with exactly \( k \) points on every line and \( g \) points in every class, in particular in case \( v = kg \), i.e. in transversal designs \( TD[k,g] \)), then each point of an oval is on exactly one tangent. The number \( t_B(x) \) of tangents through a point \( x \in B \) is odd iff \( r = r_p = |B| \) is odd, and even otherwise. In a Steiner system \( S(2,k;v) \) it is well known (e.g. Hall [5]) that

\[
(1.1) \quad r = \frac{v-1}{k-1}, \quad b = \frac{vr}{k} = \frac{v(v-1)}{k(k-1)}.
\]

There is a huge literature on Steiner systems \( S(t,k;v) \), see the book [9] edited by Lindner and Rosa, in particular the bibliography by Doyen and Rosa (in this book [9]) with more than 700 titles. In case \( k = 3 \) we get a Steiner triple system (STS) with \( r = (v-1)/2, b = v(v-1)/6 \). Let \( STS \) be the set of \( v \in \mathbb{N} \) for which an \( STS(v) \) exists. It is well known that \( STS = 6\mathbb{N}_0 + \{1,3\} \).

In this paper we shall show that for each \( v = 3 \) or \( 7 \mod 12 \) there are STS's with hyperovals, for each \( v \in STS \) there are STS's with ovals, and for almost all \( v \in STS \) there are STS's without ovals. The proof of the last two assertions was considerably improved by several remarks of W. Piotrowski [12].