Let $A = (A,F)$ be an algebra. $F$ is allowed to have both finitary and infinitary operations. A subset $T \subseteq A$ is called a stock or trunk of $A$ if for each $f \in F$ of arity $\alpha$, $f(a_i : i < \alpha) \in T$ whenever $a_i \in T$ for at least one $i < \alpha$. The empty set is a stock and if $F$ contains no nullary operations, then every stock is a subalgebra of $A$. We say that $z \in A$ is a distinguished element of $A$ if \{z\} is a stock of $A$. If $F$ contains an operation with arity at least two, then the distinguished element is unique, if it exists.

**PROPOSITION 1.** The set of all stocks of $A$ forms a complete sub-lattice of the Boolean algebra of all subsets of $A$.

**Proof.** Trivial.

We associate with each stock $T$ of $A$ a congruence $C(T)$ defined by $x C(T) y$ if and only if $x = y$ or $x,y \in T$. It is easy to see that $C(T)$ is a congruence, which we call the Rees congruence generated by $T$ [8]. The only quotient algebras that we consider in this paper are of the form $A/C(T)$. We write $A/T$ for $A/C(T)$ which can be identified with $(A - T) \cup \{z\}$ where $z$ is the distinguished element of $A/T$.

The next two propositions are versions of the second and third isomorphism theorems. Their proofs are omitted.

**PROPOSITION 2.** Let $T$ be a stock of $A$ and $S = (S,F)$ a subalgebra then

i) $T = (T \cup S,F)$ is a subalgebra of $A$,

ii) $T$ is a stock of $T$ and $T \cap S$ is a stock of $S$,

iii) $T/T \cong S/(T \cap S)$.

**PROPOSITION 3.** Let $K$ be a stock of $A$ and let $h : A \to A/K$ be the natural isomorphism. Then $h$ induces a one-to-one correspon-
between the lattice of all stocks of $A$ which contain $K$ onto
the lattice of all non-empty stocks of $A/K$. If $P \supseteq K$ is a stock,
then $(A/K)/(P/K) \cong A/P$.

For $x \in A$ let $J(x)$ be the stock generated by $x$. Define an
equivalence relation $J$ by $x J y$ if $J(x) = J(y)$. For $r \in A$
define $J_r = \{ x \in A : J(x) = J(r) \}$ and set $I(r) = J(r) - J_r$ (note
$J_r \subseteq J(r)$).

Proposition 4. (i) $I(r) = \{ x \in A : J(x) \not\subseteq J(r) \}$ (ii) $I(r)$
is a stock of $A$ maximal in $J(r)$.

Proof. The proof of (i) is easy. To prove (ii) we need to show
$I(r)$ is a stock. If it were not there would be a $t \in I(r)$ and
$f \in F$ with $z = f(a_i : i < a) \notin I(r)$ for some $\{ a_i : i < a \}$ such
that $t = a_i$ for some $i$. Since $t \in J(r)$, $z \in J(r) - I(r)$. Hence
$J(z) = J(r)$, so $t \in J(z)$. Clearly $z \in J(t)$. Therefore $J(t) = 
J(z) = J(r)$, contradicting $t \in I(r)$.

Let $B$ be a subset of $A$. We call $J(x)/I(x)$, $x \in B$ a
principal factor of $A$ over $B$. Let $R$ and $P$ be stocks of $A$
with $P \nsubseteq R$. We call a finite strictly decreasing chain

$$
(1) \quad R = S_0 \supseteq S_1 \supseteq \cdots \supseteq S_k = P
$$

of stocks of $A$ a $G$-principal series of $A$ from $R$ till $P$ if each
$S_i$ is maximal in $S_{i-1}$. The algebras $S_{i-1}/S_i$ are called the
factors or quotients of the series. If

$$
(2) \quad R = T_0 \supseteq T_1 \supseteq \cdots \supseteq T_n = P
$$

is another $G$-principal series from $R$ till $P$ we say that (1) and
(2) are isomorphic if $k = n$ and there is a permutation $\pi$ on
$\{0,1, \ldots, k-1\}$ so that $S_i/S_{i+1} \cong T_{\pi(i)}/T_{\pi(i)+1}$.

Theorem 1. Let $A = (A,F)$ be any algebra. Let $(R,P)$ be a
pair of stocks of $A$ which admits a $G$-principal series (1). The
factors of (1) are isomorphic (taken in a certain order) to the
principal quotients over $R-P$. In particular, any two $G$-principal
series from $R$ till $P$ are isomorphic.

Proof. We begin with any factor of (1), $S_i/S_{i+1}$,
$i \in \{0,1, \ldots, k-1\}$. Let $m \in S_i - S_{i+1}$, then $J(m) \cup S_{i+1}$ is a
trunk of $A$ by proposition (1), so that $m$ belongs to it and it