P.M. Cohn introduced in [4] the inversive localization at a semiprime ideal $N$ of a left Noetherian ring $R$. He gave a construction for a ring of quotients $R_T(N)$ universal with respect to the property that every matrix regular modulo $N$ is invertible over $R_T(N)$. That is, in each ring $(R_T(N))^n$ of $n \times n$ matrices over $R_T(N)$, every element of $(R)^n$ which is regular modulo $(N)^n$ becomes invertible. The ring $R_T(N)$ always exists, but it can be very difficult to determine. In fact, it is hard to compute even the kernel of the mapping $R \to R_T(N)$. On the other hand, $R_T(N)$ has some very desirable properties which are lacking in the torsion theoretic localization $R_C(N)$, and so it appears to be worthy of further study. This paper contains the announcement of some preliminary results in studying inversive localization. It also contains some explicit computations, since one of the first tasks must be to build a collection of examples. Included is the computation of $A_T(P)$ for every prime ideal $P$ of the ring of formal matrices $A = \begin{bmatrix} R & M \\ N & R \end{bmatrix}$, where $R$ is a commutative ring and $M$ and $N$ are modules over $R$ which have the pairings necessary to define matrix multiplication in $A$. This includes as special cases several examples given by Cohn in [4].

§1. Some properties of the inversive localization

The ring $R$ is assumed to be an associative ring with identity, and all modules are assumed to be unital. If $R$ is left Noetherian and $N$ is a semiprime ideal of $R$, then the ring $R_T(N)$ is constructed as follows (see [4] and [5, p.255] for details). Let $\Gamma(N)$ be the set of all square matrices over $R$ which are regular modulo $N$. For each $n \times n$ matrix $\gamma = (a_{ij}) \in \Gamma(N)$ take a set of $n^2$ symbols $(a_{ij}) = \gamma'$, and take a ring presentation of $R_T(N)$ consisting of all of the elements of $R$, as well as all of the elements $a_{ij}'$ as generators; as defining relations take all of the relations holding in $R$, together with the relations, in
matrix form, $\gamma \gamma' = \gamma' \gamma = 1$, for each $\gamma \in \Gamma(N)$. The mapping $\lambda : R \to R_{\Gamma(N)}$ is an epimorphism in the category of rings, and $R_{\Gamma(N)}/J(R_{\Gamma(N)})$ is the classical ring of quotients of $R/N$, under the embedding $\lambda' : R/N \to R_{\Gamma(N)}/J(R_{\Gamma(N)})$ induced by $\lambda$. (The Jacobson radical of the ring $R$ will be denoted by $J(R)$.) The latter property will be used in Theorem 1.1 to characterize $R_{\Gamma(N)}$.

The ring $R_{\Gamma(N)}$ can be constructed in certain cases even when $R$ is not left Noetherian. In fact, Cohn's proofs remain valid when $N$ is any semiprime ideal such that the factor ring $R/N$ is a left Goldie ring (this ensures the existence of the classical ring of quotients $Q_{\text{cl}}(R/N)$). A semiprime (prime) ideal which satisfies this condition will be called a semiprime (prime) Goldie ideal. Working in this generality means that the inversive localization can be defined, for example, at any prime ideal of a ring with polynomial identity.

If $N$ is a semiprime Goldie ideal of the ring $R$, consider the following conditions on a ring $S$ and ring homomorphism $\phi : R \to S$. Note that any ring which satisfies these conditions must be unique (up to isomorphism).

$J_1$. The homomorphism $\phi$ induces a ring homomorphism $\phi' : R/N \to S/J(S)$ such that the following diagram commutes. (The mappings $R \to R/N$ and $S \to S/J(S)$ are the natural projections.)

$$
\begin{array}{ccc}
R & \xrightarrow{\phi} & S \\
\downarrow & & \downarrow \\
R/N & \xrightarrow{\phi'} & S/J(S)
\end{array}
$$

$J_2$. The ring $S/J(S)$ is a classical ring of quotients of $R/N$, under the embedding $\phi' : R/N \to S/J(S)$.

$J_3$. If $\theta : R \to T$ is a ring homomorphism which satisfies conditions $J_1$ and $J_2$, then there exists a unique ring homomorphism $\theta^* : S \to T$ such that the following diagram commutes.

$$
\begin{array}{ccc}
R & \xrightarrow{\phi} & S \\
\theta & \searrow & \theta^* \\
& & T
\end{array}
$$