Let $\mathcal{L}(\mathcal{H})$ denote the algebra of all bounded linear operators acting on a complex separable infinite dimensional Hilbert space $\mathcal{H}$. C. Foiaş, C. Pearcy and D. Voiculescu [3] proved that the subset

$$(BQT)_{qs} = \{ A \in \mathcal{L}(\mathcal{H}) : A \text{ is quasisimilar to some biquasitriangular operator} \}$$

is (norm) dense in $\mathcal{L}(\mathcal{H})$ and asked whether $(BQT)_{qs}$ is actually equal to $\mathcal{L}(\mathcal{H})$.

The answer is negative. Furthermore, suitable modifications of the Apostol-Morrel models [2] provide the following result

**THEOREM A.** The complement of $(BQT)_{qs}$ is also dense in $\mathcal{L}(\mathcal{H})$.

The main ingredient of the proof is the construction of large family of operators with very particular properties. Let $\Omega$ be a nonempty bounded connected open subset of the plane such that $\partial \Omega$ consists of finitely many pairwise disjoint regular analytic Jordan curves and let $\Lambda = \{ \lambda_1, \lambda_2, \ldots, \lambda_m \}$ be a finite subset of $\mathbb{C} \setminus \Omega$ having exactly one point in each component of this set. Let $\varepsilon > 0$ be small enough so that $\Lambda \cap (\Omega + \varepsilon t) = \emptyset$ for $0 < t \leq 1$ and define $\Gamma = \{(z,t) \in \mathbb{C} \times (0,1) : z - \varepsilon t \in \partial \Omega \}$. (The "leaning tower".) Let $W^{2,2}(\Gamma)$ be the Sobolev space of all distributions on the analytic manifold $\Gamma$ whose partial derivatives up to order 2 belong to $L^2(\Gamma, dm)$ ($dm$ is the "area measure" induced by 3-dimensional Lebesgue measure).

$W^{2,2}(\Gamma)$ can be identified with a Banach algebra (under an equivalent norm) of continuous functions on $\Gamma$ under pointwise operations [1].

This algebra contains a subalgebra

$A^{2,2}(\Gamma) = \text{closure } \{ f(z,t) = \sum_{k=0}^{n} t^k f_k(z) : n = 1,2,\ldots \}$

consisting of all "analytic elements" of $W^{2,2}$. (Here $f_k$ denotes an arbitrary rational function of $z$ with poles in $\Lambda$.)
Let $T \in \mathcal{L}(W^2, \Omega)$ be the operator "multiplication by $z$" ($T_f(z,t) = zf(z,t)$).

Clearly, $A^2, \Omega(T)$ is invariant under $T$ and (1) $a'(T)$ (the commutant of $T$) $\supseteq \{M_g : g \in W^2, \Omega\}$ (where $M_g = "\text{multiplication by } g")$ and (2). If $L = T|A^2, \Omega(\Gamma)$, then $a'(L) \supseteq \{M_g : g \in A^2, \Omega(\Gamma)\}$. Let $\sigma(T)$, $E^r(T)$ and $E^l(T)$ denote the spectrum, the left essential spectrum and the right essential spectrum of $T$, respectively.

**THEOREM B.** With the above notation:

(i) $\mathcal{M}[W^2, \Omega(\Gamma)]$ (Gelfand spectrum) $= \Gamma$.

(ii) $\mathcal{M}[A^2, \Omega(\Gamma)] = \{(z,t) \in C \times [0,1] : z - st \in \Omega^+\}$.

(iii) $\sigma(T) = E^r(T) = E^l(T) = E^l(L) = \{z : (z,t) \in \Gamma\}$.

(iv) $\sigma(L) = E^r(L) = \{z : (z,t) \in \mathcal{M}[A^2, \Omega(\Gamma)]\}$.

(v) $\operatorname{Ker}(\lambda - L) = \{0\}$ and $\dim \operatorname{Ker}(\lambda - L)^* = \infty$ for every $\lambda \in \sigma(L) \setminus E^l(L)$.

(vi) If $e(z,t) \equiv 1$, then $A^2, \Omega(\Gamma) = a''(L)e = \{Ae : A \in a''(L)\}$, where $a'(L)$ and $a''(L)$ denote the commutant and the double commutant of $L \in \mathcal{L}(A^2, \Omega(\Gamma))$, resp. This implies $a''(L) = a'(L)$ is a maximal abelian strictly cyclic subalgebra of $\mathcal{L}(A^2, \Omega(\Gamma))$ with strictly cyclic vector $e$.

(vii) If $L'$ is quasisimilar to $L$, then $L'$ is actually similar to $L$.

**Proof.** (i)-(v) are standard facts.

(vi) Let $A \in a'(L)$ and let $\eta, \tau \in [0,1], \eta \neq \tau$.

Assuming that $|\eta - \tau| > \beta \delta > 0$, let $H_\eta(z,t) = H_\eta(t) \in A^2, \Omega(\Gamma)$ be defined by