Various additive formulae for supports and singular supports of convolutions are studied in terms of the Fourier transform in complex domain. Part of the material presented below was announced in our note [4]. The authors wish to thank Professor L. Hörmander for his most helpful comments.

§ 1 - Notation and Auxiliary Facts -

Throughout this note we shall use the standard notation of the theory of distributions (cf. [22, 15]). In particular, $\mathcal{E}' = \mathcal{E}'(\mathbb{R}^n)$ is the convolution algebra of distributions with compact support in $\mathbb{R}^n$. For $\varphi \in \mathcal{E}'$, $\hat{\varphi}$ denotes the distribution symmetric to $\varphi$, and $\hat{\varphi}$ is the Fourier transform of $\varphi$, i.e. $\hat{\varphi}(\xi) = \varphi(e^{-i\langle x, \xi \rangle})$ where $\xi = \xi + i\eta \in \mathbb{R}^n$ and $\langle x, \xi \rangle = \sum_{j=1}^{n} x_j \xi_j$. We write $\omega(\varphi) = \log(2 + \|\varphi\|)$, $\varphi \in \mathbb{R}^n$. The convex hull of the set $\text{supp} \; \varphi$ (sing supp $\varphi$ resp.) will be denoted by $[\varphi]$ ($[\varphi]$ resp.).

If $A$ and $B$ are two subsets of $\mathbb{R}^n$, $A \pm B$ means the set of all points $x \pm y$ for $x \in A$ and $y \in B$. Similarly,

\begin{equation}
A + B \subseteq C \Rightarrow A \subseteq C - B,
\end{equation}

the converse being obviously false.

Let $\mathcal{K}$ be the class of all compact sets in $\mathbb{R}^n$.

For each $K \in \mathcal{K}$, set

\begin{equation}
\varphi_K(\xi) = \sup_{x \in K} \langle x, \xi \rangle \quad (\xi \in \mathbb{R}^n).
\end{equation}

Hence $\varphi_K = -\infty$. Let $\mathcal{H}$ be the class of all support functions in $\mathbb{R}^n$, i.e. the class consisting of the constant function $-\infty$ and all finite (hence continuous) func-
tions \( h(\xi) \) defined on \( \mathbb{R}^n \) such that

\[
(3) \quad h(c\xi) = c \cdot h(\xi) \quad (c \geq 0) ; \quad h(\xi_1 + \xi_2) \leq h(\xi_1) + h(\xi_2) .
\]

Formula (2) defines a mapping \( I : K \rightarrow h_K \) of the class \( \mathcal{K} \) into \( \mathcal{K} \). Conversely, if we set for every \( h \in \mathcal{K} \),

\[
(4) \quad K_h = \{ x : \langle x, \xi \rangle \leq h(\xi), \forall \xi \} ,
\]

then \( J : h \rightarrow K_h \) is a mapping of \( \mathcal{K} \) into \( \mathcal{K} \) such that \( IoJ \) and \( JoI \) are identity mappings of classes \( \mathcal{K} \) and \( \mathcal{K} \) respectively (cf. [21]). Moreover, this natural correspondence between \( \mathcal{K} \) and \( \mathcal{K} \) is in a certain sense linear and positive:

**Lemma 1**: Given \( K_1, K_2 \in \mathcal{K} \) and \( a \geq 0 \), we have

\[
(5) \quad K_2 = a K_1 \quad \text{iff} \quad h_{K_2} = a h_{K_1} ,
\]

and

\[
(6) \quad K_3 = K_1 + K_2 \quad \text{iff} \quad h_{K_3} = h_{K_1} + h_{K_2} .
\]

Moreover, \( K_1 \subseteq K_2 \) iff \( h_{K_1} \leq h_{K_2} \) and for some \( \xi_0 \in \mathbb{R}^n \)

\[
h_{K_1}(\xi_0) < h_{K_2}(\xi_0).
\]

The proof of this lemma is simple and can be found, e.g., in [21].

**Corollary 1**: Let \( A_1, A \) and \( B \) be non-empty sets in \( \mathcal{K} \) such that

\[
A \subseteq A_1 \quad \text{and} \quad A_1 + B \subseteq A + B . \quad \text{Then} \quad A_1 = A .
\]

If \( \Omega \) is an open convex set in \( \mathbb{R}^n \), one can still define its support function \( h_\Omega \) by formula (2); the function \( h_\Omega \) satisfies (3), but for \( \Omega \) unbounded, \( h_\Omega \) is only lower-semicontinuous, and for some values of \( \xi \), \( h_\Omega(\xi) = +\infty \).

**Lemma 2**: Let \( K \in \mathcal{K} \) and \( \Omega \) be a convex open set such that

\[
\emptyset \neq K \subset \Omega \subset \mathbb{R}^n . \quad \text{Then, for some} \quad x_0 \in \partial K, y_0 \in \partial \Omega, \quad \text{and} \quad |\xi| = 1 ,
\]