1. Introduction

Accurate long term solutions to certain satellite orbit integration problems may be obtained by revolution skipping methods. When many revolutions are similar, the trajectory can be extrapolated several revolutions ahead, without significant loss of accuracy.

These methods require that the change in the elements over one revolution be computed using a reliable orbit integration method. This finite change, due to secular and long period perturbations, is used to extrapolate the orbit $M$ revolutions ahead. Another single revolution integration is performed and the process is repeated. In this way only one (or in some cases two) out of $M$ revolutions are actually integrated. Since the computational work in the extrapolation formula is small, much computer time can be saved. The two-stage algorithm of external extrapolation and internal (single revolution) integration is called "multirevolution integration." The extrapolation algorithm is similar to that for the classical multistep methods for differential equations. In fact, the multirevolution methods are a generalization of the classical methods.

The finite change in the elements over one revolution defines a first order nonlinear difference equation

\[ \Delta \vec{y}_p = \vec{f}(\vec{y}_p, \rho) \]

where $\vec{y}_p$ is a vector of elements that completely defines the orbit and
\[ \Delta \gamma = \gamma_{p+1} - \gamma_p. \]  

The independent variable is the revolution number, denoted by \( p \). The revolutions are counted from any reference point on the orbit, such as apogee, perigee or node. The orbit integration problem has now been transformed from that of solving a system of first order nonlinear differential equations to that of solving a system of first order nonlinear difference equations. There is the disadvantage that the right hand sides of the difference equations cannot be written down in terms of elementary functions. This prevents any attempt at an analytical solution and suggests a numerical solution such as that provided by the multirevolution methods. The solution of Eq. (1.1) provides the osculating orbital elements at any reference point, i.e., \( \gamma = 0, 1, 2, \ldots \).

The earliest investigators of the multirevolution methods were Taratynova (1961), Mace and Thomas (1960), and Cohen and Hubbard (1960). The predictor formula given by Mace and Thomas is a generalization of the Adams-Bashforth method, and the corrector formula given by Cohen and Hubbard is a generalization of the Adams-Moulton method. Other derivations of the predictor-corrector formulae are given by Boggs (1968) and Velez (1970). Boggs also gives a suitable method for starting the multirevolution integration and discusses modifications of the basic algorithm.

2. Multirevolution Predictor-Corrector Formulae

Without loss of generality, scalar difference equations may be considered. Then Eqs. (1.1) and (1.2) become

\[ \Delta \gamma = f(y_p, p), \]  
\[ \Delta \gamma = \gamma_{p+1} - \gamma_p, \]

and \( p \) is an integer. Let \( M \) be the number of revolutions skipped. Then the dependent variable \( y \) will be defined on a series of large and small grid, the large grid being separated by \( M \) small grid. Let \( n \) be the large grid number, then \( f \) is given at the large grid by