1. INTRODUCTION

In this paper an equivalence between solutions of collocation methods and fixed points of Iterated Defect Correction (IDeC) methods is proved. Therefore the IDeC-methods can be regarded as efficient schemes for solving collocation equations. Attention is restricted to the application of the IDeC to ordinary differential equations (initial value problems and two point boundary value problems). Extension to other types of operator equations (e.g. partial differential equations, integral equations,...) is straightforward.

In Section 2 special variants of collocation methods, which are of importance in connection with the IDeC are discussed. The basic ideas behind the IDeC are presented in Section 3. The equivalence between collocation schemes and the fixed points of the IDeC-methods is established in Section 4.

2. COLLOCATION METHODS

2.1. Collocation methods for two point boundary value problems

We consider problems of the form

\[(2.1a) \quad y' = f(t,y), \quad t \in [a,b]\]
\[(2.1b) \quad g(y(a),y(b)) = 0\]

where y, f and g are vector-valued functions of dimension n with f and g sufficiently smooth. A number of papers about collocation methods applied to (2.1) have appeared recently in the literature on the numerical solution of BVPs for ODEs (e.g. de Boor, Swartz [3], Russel, Shampine [9], Weiss [11]). From the class of collocation schemes, we consider the following special type (cf. Weiss [11]):

The collocation solution is a continuous piecewise polynomial which satisfies (2.1a) at given (collocation) points.

We now introduce the notation to be used below. The grid is given by
We consider the space of continuous piecewise polynomial functions \( P(t) \) [vector-valued of dimension \( n \)] defined by

\[
P(t) := \begin{cases} 
P_i(t), & t \in [t_i, t_{i+1}), 
\end{cases} 
i = O(1)I-1
\]

(2.3)

where all polynomials \( P_i \) are of degree \( m \). On (2.2) we construct the sub-grid

\[
t_i, k := t_i + \xi_k H_i, \quad i = O(1)I-1, \quad k = 1(1)m
\]

(2.4)

with collocation nodes

\[
O < \xi_1 < \ldots < \xi_m < 1
\]

(2.5)

(important special cases satisfying (2.5) are the Gauss-Legendre, the Lobatto and the Radau points). The collocation equations become

\[
P'_i(t_{i,k}) = f(t_{i,k}, P_i(t_{i,k})), \quad i = O(1)I-1, \quad k = 1(1)m.
\]

(2.6)

If \( \xi_1 = 0 \) or \( \xi_m = 1 \) in (2.5), then \( P'_i(t_{i,1}) \) or \( P'_i(t_{i,m}) \) is interpreted as the right derivative or the left derivative, respectively. If \( \xi_1 = 0 \) and \( \xi_m = 1 \), then two collocation equations (2.6) hold at every gridpoint \( t_i = t_{i,1} = t_{i-1,m} \). Together with the boundary condition

\[
g(P_0(a), P_{I-1}(b)) = 0
\]

(2.7)

and the continuity conditions

\[
P_i(t_{i+1}) = P_{i+1}(t_{i+1}), \quad i = O(1)I-2,
\]

(2.8)

the collocation conditions yield \( n \cdot I \cdot (m+1) \) equations for the \( n \cdot I \cdot (m+1) \) unknown coefficients of \( P \).

2.2. Collocation methods for initial value problems

The method of Section 2.1 can be interpreted as a method for solving IVPs, if in (2.1) the boundary condition is replaced by

\[
g(y(a), y(b)) = y(a) - y_a = 0.
\]

(2.9)

In this situation, it is possible to solve the equations (2.6) block by