0. Introduction

We are interested in solving the inverse eigenvalue problem associated with band matrices. We shall place particular emphasis on giving a stable numerical algorithm for determining such matrices.

Let $A$ be a real, symmetric matrix with

$$a_{ij} = 0 \text{ when } |i - j| \geq p.$$ 

Define

$$(A^{(k)})_{i,j} = a_{ij} \quad \begin{cases} i = k, k+1, \ldots, n; \\ j = k, k+1, \ldots, n \end{cases},$$

so that $A^{(k)}$ is derived from $A$ by striking out the first $(k-1)$ rows and columns of $A$. We denote the eigenvalues of $A^{(k)}$ as $\{\lambda_i^{(k)}\}_{i=1}^{n-k+1}$ and, we assume

$$\lambda_1^{(k)} < \lambda_{i+1}^{(k)} \quad (i = 1, 2, \ldots, n-k+2)$$

and

$$\lambda_1^{(k)} < \lambda_1^{(k+1)} < \lambda_{i+1}^{(k)}.$$ 

The problem then is to determine the matrix $A$ from the $p$ sets of eigenvalues $\{\lambda_i^{(k)}\}_{i=1}^{n-k+1}$, $(k = 1, 2, \ldots, p)$. Note that the number of unknowns is precisely equal to the number of eigenvalues specified.

For $A$ tri-diagonal ($p = 2$), the problem has been studied extensively (cf. [1], [2], [3]). The numerical algorithm derived in this paper is similar to that given in [3] but the method for deriving the algorithm is different.

In order to simplify the notation and discussion, we shall primarily concern ourselves with five diagonal matrices in this paper and discuss the general case in

*The work of this author was supported in part by the NSF MCS75-13497-A01 and in part by ERDA EY-76-3-03-0326PA#30.
a future paper. We use the following notation. Let

$$K \equiv A = \begin{bmatrix}
  a_1 & b_1 & c_1 \\
  b_1 & c_2 & b_2 & c_2 \\
  & c_1 & b_2 & \cdots & c_{n-2} \\
  & & c_2 & \cdots & \cdots & b_{n-1} \\
  & & & \cdots & c_{n-2} & b_{n-1} \\
  & & & & c_{n-2} & a_n
\end{bmatrix},$$

we assume the integer $n$ is even. Let the eigenvalues of $K, L, M$ be $(\kappa_i)_{i=1}^n, (\lambda_i)_{i=1}^{n-1}, (\mu_i)_{i=1}^{n-2}$, respectively. We assume

$$\kappa_i < \kappa_{i+1}, \quad \lambda_i < \lambda_{i+1}, \quad \mu_i < \mu_{i+1}$$

and

$$\kappa_i < \lambda_i < \kappa_{i+1}, \quad \lambda_i < \mu_i < \lambda_{i+1}.$$

In section 1, we give the solution of a related problem which is required for the final solution. The block Lanczos algorithm is described in section 2 since it plays a fundamental role in generating $K$. Finally in section 3, we describe the method for generating $K = A$.

1. Solution of Related Problem

In order to determine $K$ from its eigenvalues we need the solution to a related problem. The solution to this related problem is derived in [4] and we summarize it here.

Let $A$ be a real symmetric matrix with distinct eigenvalues $(\alpha_i)_{i=1}^n$ and let $Q$ be the matrix of eigenvectors.