**Introduction**

Let $M$ be a differentiable manifold of dimension $m$. A field of $(m-q)$-planes on $M$ is a vector subbundle $\eta$ of rank $m-q$ of the tangent bundle $\tau M$ of $M$. This field is said to be completely integrable if it defines a foliation. This means that for each point $x$ of $M$, there is a submersion $f$ of a nbhd $U$ of $x$ in $\mathbb{R}^q$ such that the kernel of the differential $\tau f$ at any point $y$ of $U$ is the fiber of $\eta$ at $y$.

We shall denote by $v(\eta)$ the quotient $\tau M/\eta$ which is a vector bundle of rank $q$ on $M$; let $\pi$ be the natural epimorphism $\tau M \to v(\eta)$.

Let us compare the following two theorems.

**Gromov-Phillips transversality theorem**

Let $\eta$ be a field of $(m-q)$-planes on $M$. A map of a differentiable manifold $X$ in $M$ is said to be transversal to $\eta$ if the composition $\pi \cdot \tau f : \tau M \to v(\eta)$ is an epimorphism of vector bundles, namely for each point $x$ of $X$ it is a linear surjective map of the fiber $\tau_x M$ on the fiber $v_{\tau_x} (\eta)$. Note that if $\eta$ is completely integrable, so is the subbundle $\text{Ker} \ (\pi \cdot \tau f)$ of $\tau X$.

If $X$ is an open manifold (i.e. if $X - \partial X$ has no compact connected component), the theorem asserts that a map $f : X \to M$ is homotopic to a map transversal to $\eta$ iff $f$ can be covered by an epimorphism $\tau X \to v(\eta)$. Moreover two maps $f_0$ and $f_1$ of $M$ in $X$ which are transversal to $\eta$ are homotopic through a differentiable family of maps transversal to $\eta$ iff $\pi \cdot \tau f_0$ and $\pi \cdot \tau f_1$ are homotopic through
Bott theorem

Let $\eta$ be a completely integrable field of $(m-q)$-planes on the $m$-dimensional manifold $M$. Then any rational cohomology class of $M$ of degree $> 2q$ which is a polynomial in the Pontrjagin classes of $v(\eta)$ is always zero.

These two statements reveal two opposite phenomena. In the first case, the condition for a map to be transversal to $\eta$ is given by an open differential inequality on its differential; the obvious necessary condition on the vector bundles is then sufficient to get, after an homotopy, a map transversal to $\eta$ (if $X$ is open). On the other hand, the condition for a field $\eta$ to be completely integrable is given by a differential equality. Bott theorem shows that in order that a field of $(m-q)$-planes be homotopic to an integrable one, very strong necessary conditions must be satisfied.

In these lectures, we shall try to give a general frame for the discussion of homotopy problems involving integrability conditions.

Before giving the general abstract definition of a $\Gamma$-structure, we have to give a few words of justification.

Let $G$ be a pseudogroup whose elements are homeomorphisms of open sets of Euclidean space $\mathbb{R}^q$. For instance, the elements of $G$ could be all diffeomorphisms of class $C^r$, or those which are volume preserving, or complex analytic local automorphisms of $C^n = \mathbb{R}^q$, when $q = 2n$.

On a topological manifold $X$, many structures considered in differential geometry are defined by an atlas $A$ compatible with $G$: namely, the elements of $A$ are homeomorphisms $f_i$ of open sets