From the experimental point of view probability enters quantum theory just like classical statistical physics, i.e. as an expected relative frequency. However it is well known that the statistical formalism of quantum theory is quite different from the usual Kolmogorovian one involving, for example, complex numbers, amplitudes, Hilbert spaces ... . The quantum statistical formalism has been described, developed, applied, generalized with the contributions of many authors; however its theoretical status remained, until recently, quite obscure, as shown by the widely contrasting statements that one can find in the vast literature concerning the following questions.

**Question I.** Is it possible to justify the choice of the classical or the quantum statistical formalism, for the description of a given set of statistical data, on rigorous mathematical criteria rather than on empirical ones?

In particular, is the quantum statistical formalism in some sense necessary, or (as some authors seem to believe) is it an historical accident and the whole quantum theory can be developed within the framework of the classical Kolmogorovian model?

**Question II.** If (as it will be shown to be the case) it is possible to device a rigorous mathematical criterion which allows to discriminate between the two statistical models, then the Kolmogorovian model must include a hidden postulate which limits its applicability to the statistical description of the natural phenomena. Which of the Kolmogorov axioms does it play, for probability, the role played by the parallel axiom for geometry?

**Question III.** Which new physical requirements should substitute the hidden axiom of the Kolmogorovian model mentioned in Question II.? Will such requirements be sufficient to account for all the specific features of the quantum statistical model?

**Question IV.** Are the Kolmogorovian, the usual quantum model, and its known generalizations, the only statistical models which can arise in the description of nature?

In the present paper we describe how the above mentioned problems can be formulated and solved in a rigorous mathematical way. In particular the negative answer to Question IV.), strongly supports the point of view that, in analogy with geometry, one should look at probability theory not as the study of the laws of chance but of the possible, mutually inequivalent models for the laws of chance,
the choice among which for the description of natural phenomena, being a purely experimental question. The main goal of quantum probability should then be recognized in the inner development of these models, their classification, the analysis of their mutual relations, and in no case reduced to a translation of classical probabilistic results into a quantum (or "non-commutative") language. On the contrary, the deepest problems of quantum probability are just those in which this translation becomes impossible, as a consequence of the different axioms which lie at the foundation of the two theories. For lack of space our discussion will be limited to Questions I.), ... , IV.) and to the problems which arise in connection with them. In particular we will not discuss the relevance of the answers to these questions for the old interpretational problems of quantum theory (cf.[7]), nor the important steps that have been made in the last years towards the inner development of the usual quantum probabilistic model (for these we refer to the papers in these proceedings). We believe that once understood the origins and the meaning of quantum probability, it will be easier to go further along the lines of the analysis of more general classes of nonkolmogorovian probabilistic models the deepening of the usual quantum quantum one, as well as the study of the connections between the kolmogorovian and the quantum model.

In sections (1.) and (2.) we review some known results concerning the answer to Question I.). Question II.) has been dealt with extensively in [7] and will not be discussed here; in sections (3.) and (4.) we answer Question III.) giving the proofs of some results announced in previous papers [5], [6]; finally in sections (5.) we outline a geometrical generalization of the quantum probabilistic formalism which, in view of some recent progresses in theoretical physics and in the theory of operator algebras, seems to be the most promising line along which to investigate the answer to Question IV.).

1. The statistical invariants.

The answer to Question I.) of § (0.) is based on the following idea: the existence of a Kolmogorovian (resp. a quantum) model which describes a given set of statistical data imposes some constraints on them which can be explicitly computed. It happens that in nature one can find some sets of statistical data satisfying the constraints which characterize the existence of a quantum statistical model but not those characterising the existence of a Kolmogorovian model (or conversely!). This proves that, as long as we want to describe these statistical data within a single mathematical model, giving up the classical Kolmogorovian model is a mathematical and experimental necessity. No new experiment is needed to prove this statement: it is sufficient to apply the results formulated in the following to the oldest and well established data of quantum theory. Locally (i.e. when restricted to statistical data concerning sets of compatible observables) the quantum statistics reduces to the classical one: the really new features of quantum probability arise in the description of statistical data concerning mutually incompatible sets of observables.

The following notations will be used throughout the paper: let \( T \) be a set (index set); let, for each \( x \in T \), be given an observable quantity \( \mathbf{A}(x) \) whose values will be denoted \( a_1(x), \ldots, a_n(x) \). Unless explicitly stated \( n \) will be a finite positive integer independent on \( x \in T \). Heuristically \( \mathbf{A}(x) \) should be thought as a complete set of compatible observables. For each \( x \in T \) and \( a, \beta = 1, \ldots, n \), let us denote...