1.- INTRODUCTION

The essential role played by the Nijenhuis tensor field $N$ in the problem of the integrability of an almost complex structure $J$ is well known. The aim of the present paper is to point out further properties of the tensor field in the case when the manifold $V$ is endowed with an almost hermitian structure.

An interesting relation linking the Nijenhuis field $N$ and the field $DJ$, deduced from $J$ by covariant differentiation in the riemannian connection, is obtained in Sec. 4 (Th.1).

This relation is a very useful tool to prove a series of theorems.

A new proof of the known fact, that $G_1$-spaces and $G_2$-spaces can be defined in terms of $N$ only, is given in Sec. 6 (Th.3).

A characterization of hermitian manifolds is also obtained in Sec. 6 (Th. 2). This result points out that there is a sort of analogy between these manifolds and $G_1$-spaces.

Finally, some known classes of almost hermitian manifolds can be characterized, by assigning particular expressions in terms of $DJ$ to the tensor field $N$ (Th. 4, Th. 5, Th. 6; Sec. 7).

2.- ISOMORPHISMS $\alpha$, $W$, $\lambda$, $\gamma$

Let $V$ be an almost hermitian manifold of dimension $2n$ and class $c^{2n+1}$ (1).

(1) For the basic facts about almost hermitian manifolds see K. Yano [11], ch. 9; S. Kobayashi-K. Nomizu [5], II, ch. 9.
Let $T^r_s$ the linear space of tensor fields of type $(r,s)$ on $V$. In particular, let $g$ be the symmetric field of $T^0_2$ of class $C^1$, defining the riemannian metric on $V$ and let $J$ be the field of $T^1_1$ of class $C^{2n}$, defining the almost complex structure on $V$.

Some isomorphisms of $T^1_2$ play an essential role in the following; namely $\alpha$, $W$, $\lambda$, $\gamma$.

Let $\sigma, \varepsilon$ be the homomorphisms of symmetry, of skew-symmetry of $T^1_2$; then $\alpha = \sigma - \varepsilon$. The isomorphisms $W, \lambda$ are defined for any field $L$ of $T^1_2$ by

$$W_L = - c_3^1 (c_2^2 (L \odot J) \odot J), \quad \lambda L = c_3^1 (L \odot J) \quad (2)$$

Denote by $G$ the symmetric tensor field of $T^2_0$ satisfying $c_1^2 (g \odot G) = \delta$. Then the isomorphism $\gamma$ is defined for any field $L$ of $T^1_2$ by

$$\gamma L = c_2^1 (c_1^2 (g \odot L) \odot G)$$

Equivalent definitions of the isomorphisms $\alpha, \lambda, W$ are the following. Let $L$ be an arbitrary field of $T^1_2$; then for any $X$, $Y$ of $T^1_0$, we put

$$(\alpha L)(X,Y) = L(Y,X)$$

$$(\lambda L)(X,Y) = JL(X,Y) \quad , \quad (W L)(X,Y) = - JL(X,JY) \quad (4)$$

Similarly, the isomorphism $\gamma$ can be implicitly defined by

$$g((\gamma L(X,Y),Z) = g(L(Z,Y),X)$$

where $Z$ is an arbitrary field of $T^1_0$ and $g(\ ,\ )$ denotes inner product.

The above definitions show that the isomorphisms $W$, $\lambda$, introduced in [6], depend only on the almost complex structure $J$ and that the isomorphism $\gamma$, introduced in [7], depends only on the riemannian structure $g$.

The basic relations about the isomorphisms $\alpha$, $W$, $\lambda$, $\gamma$ are

$$\alpha \alpha = \gamma \gamma = WW = 1 \quad , \quad \lambda \lambda = - 1 \quad (1)$$

$$\alpha \lambda = \lambda \alpha \ , \ \alpha \gamma \alpha = \gamma \alpha \gamma \ , \ W \lambda = \lambda W \quad (2)$$

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(2) The symbol $\odot^r_s$ denotes contraction ([1], p. 45).

(3) $\delta$ is the classical Kronecker field of $T^1_1$.

(4) Here $J$ is regarded as an isomorphism of $T^1_0$.

(5) See [6], n. 3,5 and [7], n. 3.