CHAPTER 9
UNIQUENESS, MOMENTS, NORMALITY

Problems collected in this chapter are variations on the following theme: a "sufficiently analytic" function vanishing "intensively enough" is identically zero. The words in quotation marks get an exact meaning in accordance with every concrete situation. For instance, dealing with the uniqueness of the solution of a moment problem we often exploit traces of the analyticity of the function $\alpha \rightarrow \int x^\alpha d\mu(x)$.

The theme is wide. It encompasses such phenomena as the quasi-analyticity and the uniqueness of the moment problem, and borders on normal families (see e.g. Problem 9.5), various refinements of the maximum principle and approximation. Its importance hardly needs any explanation. The Uniqueness marks (more or less explicitly) all contents of this book. After all, every linear approximation problem (and the book abounds in such problems) is a dual reformulation of a uniqueness problem.

Every problem of this Chapter (except for 9.3 and 9.7) deals not only with "the pure uniqueness" but with other topics as well. Problem 9.1 is connected not only with zeros of some function classes but with a moment problem (as is Problem 9.2) and with Fourier - Laplace transforms of measures; in Problem 9.6 the uniqueness in analytic Gevrey classes is considered in connection with peak sets for
Hölder analytic functions. "Old" Problems 9.8 and 9.9 deal (from different points of views) with differential and differential-like operators (both have evoked a great interest, see respective commentary). Problem 9.4 has certain relation to spectral operators and to the "anti-locality" of some convolution operators (in contrast with "the locality" of convolutions discussed in 9.9). Problem 9.5 is a quantitative variation on the title theme and 9.13 gravitates towards spectral analysis-synthesis of Chapter 7. Problem 9.10 is aimed at approximation properties of exponentials and concerns also some aspects of quasianalyticity, as does Problem 9.12. Problem 9.11 deals with an interesting "perturbation" of the $\int \log |f| > -\infty$ -theorem.

The theme of this chapter emerges in some Problems of other Chapters (3.7, 4.3, 4.4, 4.9, 5.12, 7.7, 7.17, 7.18, 8.4, 10.1, 10.5, 10.6, 8.4, 8.6).