Abstract. A brief review of the classical theory of multivalued functions of a complex variable is used to introduce the classification of monogenic analytic functions by their monodromic dimension. Riemann's monodromy theorem is used to set the stage for a class of Hermite-Padé multiform approximants. The approximation problem for functions meromorphic on a Riemann surface of $m$-sheets with a finite number of singular points is completely solved. A general uniqueness of convergence theorem and another convergence theorem are given.

The subject of this paper is a possible method for the approximation of multiform functions of a complex variable. The motivation for this study lies in the applications, where one frequently wants to evaluate a function along a path between two closely spaced branch points, or on a branch cut, or even in a limit as a branch point is approached. Since there are a variety of notations currently in use for this problem, I will begin by introducing the notation [1] that I use. First I start by defining a functional element

$$f_{z_0}(z) = \sum_{n=0}^{\infty} f_n(z_0)(z-z_0)^n .$$

This Taylor series expansion about point $z_0$ is intended to converge in a disk of nonzero radius of convergence and defines a unique, single-valued, regular function of the variable $z$ in this disk. We know from

* Work performed under auspices of the US DOE.
the standard theory of analytic continuation that at any point in this
disk, say $z_1$, we may re-expand that functional element and create a
new functional element. It may be that the disk of convergence of the
new functional element includes points which were not in the disk of
convergence of the original functional element. If we repeat this
process in all possible ways we will get a function defined over some
Riemann surface. In fact only a denumerable set of steps is required
for this process as has been proven [1]. Therefore there may only be
at most a denumerable number of functional elements which are required
to complete the process of analytic continuation. Therefore at every
point in the $z$ plane which is regular after continuation along any
path from the initial functional element, there are only a finite or
at most denumerably infinite number of different coverings. Let these
coverings be

$$f_1(z; z_0), f_2(z; z_0), \ldots .$$

(2)

The various $f$'s are regular in the neighborhood of $z_0$. All the pairs

$$(z_0, w_0(1)), (z_0, w_0(2)), \ldots ,$$

(3)

constitute the monogenic analytic function generated by the original
functional element. We define in equation (3), $w_0^{(n)} = f_n(z_0; z_0)$.

Fundamental to this discussion of the theory of analytic continuation
is the following classical theorem.

**Monodromy theorem:** If $f(z)$ is regular in a simply connected
region, $G$, then $f(z)$ is uniform (single valued) there.

If we consider a region which is not simply connected, then the
situation is entirely different. As can be seen by the following
example:

$$w = z^{1/m}, \quad f_2(z) = e^{2\pi i/m} f_1(z),$$

(4)

where $m$ is a positive integer and the region is the punctured complex
plane. Here if one encircles the branch point at $z = 0$, one obtains
$f_2$ as given in equation (4) from $f_1$. Continuing in this manner we
find precisely $m$ coverings, but only one linearly independent covering
of the punctured complex plane. As a second example we consider

$$w = \ln z, \quad f_n(z) = f_1(z) + 2\pi i(n-1),$$

(5)

where $n$ is any integer. In this case there are an infinite number of
coverings of the complex plane, but only two linearly independent