1. Introduction. The first approximation and interpolation formulas for multivariable functions in the branched continued fraction form are given in [1]. These formulas are not very convenient for the practical use. At first, there is no error formula for approximation of functions by such fractions and moreover we have some troubles with the location of interpolation points. For the second, it is difficult to find an easy connection between approximants of branched continued fraction and corresponding multiple power series.

Therefore, we were looking for a particular form of branched continued fractions given in [1] which will be useful in practice. Such forms are given in [4], [8] and [9]. In this paper we deal with branched continued fractions defined in [4] and [8]. We restrict our considerations to the two-dimensional case.

By a two-dimensional continued fraction /shortly TDCF/ we mean an expression of the form

\[
K_0 + \sum_{i=1}^{\infty} \frac{a_i \cdot x y}{K_i}\] /1.1/

or

\[
\frac{a_0}{K_0} + \sum_{i=1}^{\infty} \frac{a_i \cdot x y}{K_i}\] /1.2/

where

\[K_i = b_i + \sum_{p=1}^{\infty} \frac{a_{p+1,i} \cdot x}{b_{p+1,i}} + \sum_{p=1}^{\infty} \frac{a_{i,p+1} \cdot y}{b_{i,p+1}}\]

The n-th approximant of TDCF /1.1/ is defined as

\[
f_n = \frac{A_n}{B_n} = K^{(n)}_0 + \sum_{i=1}^{[n/2]} \frac{a_i \cdot x y}{K_{i,n-2i}}\] /1.3/
while for TDCF /1.2/ as

\[ f_n = \frac{A_n}{B_n} = \frac{a_0}{b_0} + \sum_{i=1}^{\lfloor \frac{n-1}{2} \rfloor} \frac{a_i \ xy}{k_i^{\lfloor \frac{n-2i-1}{2} \rfloor}}, \quad \frac{A_1}{B_1} = \frac{a_0}{b_0}, \quad /1.4/ \]

where

\[ K_i^{\langle m \rangle} = b_i + \sum_{p=1}^{m} \frac{a_{p+1,i} x}{b_{p+1,i}} + \sum_{p=1}^{m} \frac{a_{p+1,i} y}{b_{p+1,i}}. \]

Here \( \lfloor x \rfloor \) denotes the integral part of \( x \), \( K_1^{\langle 0 \rangle} = b_i \).

TDCF-s introduced in [4] and [8] generalize in the natural way constructions based on ordinary continued fractions and can be used for solving two-variable interpolation and approximation problems. The following properties of TDCF-s can be easily checked:

/i/ Defining Property: Coefficients of the TDCF corresponding to the given two-variable power series are uniquely defined. The \( n \)-th approximant of the given TDCF is a rational two-variable function and its power series agrees with the given power series up to terms of the total power \( n \).

/ii/ Symmetry Property: The construction of TDCF corresponding to the given power series do not depend on the ordering of variables /examples of unsymmetrical branched continued fractions are given in [3] and [10] /.

/iii/ Projection Property: If we put \( x=0 \) or \( y=0 \) in TDCF /1.1/ /respectively in TDCF /1.2/ / then it reduces to the ordinary continued fraction corresponding to the reduced one variable series.

/iv/ Reciprocal Property: TDCF corresponding to the reciprocal of the given power series is the reciprocal of a TDCF corresponding to that series.

2. Convergence of TDCF. Problem of a pointwise convergence of TDCF-s leads to the investigation of convergence of TDCF-s with constant coefficients

\[ \frac{a_0}{k_0} + \sum_{i=1}^{\infty} \frac{a_i}{k_i}, \quad k_i = b_i + \sum_{m=1}^{\infty} \frac{a_{m+1,i}}{b_{m+1,i}} + \sum_{m=1}^{\infty} \frac{a_{i,m+1}}{b_{i,m+1}} \]

/2.1/

TDCF /2.1/ is said to converge if the limit of its sequence of approximants \( \lim_{n \to \infty} f_n \) exists and is finite. The value of TDCF is defined to be the limit of its sequence of approximants.