In the first part of my talk I'll recall some applications of the formalism of duality in the sense of GROTHENDIECK-VERDIER. The main results exposed here were obtained by VERDIER. Particularly we recall how it is possible to define in a very general context a Lefschetz-Number \( \text{Lef}(f,u) \), associated to the pair \((f,u)\), where

\[
f: X \longrightarrow X
\]

is an endomorphism of the scheme \( X \),

\[
u: f^*(F) \longrightarrow F
\]

is a "lifting" of \( f \) and \( F \) is a sheaf over \( X \) in a suitable topology subject to some finiteness conditions.

In fact VERDIER defines also a trace \( \text{Tr}(f,u) \) associated to the pair \((f,u)\) and proves a general Lefschetz - fixed point formula in the case of étale topology. VERDIER shows also that \( \text{Lef}(f,u) \) is a sum of local terms associated to the fixed points of \( f \), but unfortunately it is not easy to find connections between these local terms introduced by VERDIER and other known invariants.

In the particular case of schemes of dimension one GROTHENDIECK introduced other local terms and conjectures some divisibility properties about them. We will prove in the second part of this talk some weaker divisibility properties of these local terms of GROTHENDIECK.
§1. **Fibred categories.** A *fibred category* \( F \) is given by

i) a category \( \mathcal{C} \), called the **base** of \( F \)

ii) a family \( (F_X)_{X \in \text{Ob}(\mathcal{C})} \) of categories indexed by the objects of the category \( \mathcal{C} \); the category \( F_X \) is called the **fiber** of \( F \) over \( X \).

iii) a mapping \( \alpha \) which assigns to every morphism \( f : X \to Y \) in \( \mathcal{C} \) a functor \( f^\alpha : F_Y \to F_X \)

iv) a mapping \( \gamma \) which assigns to every pair \( (f,g) \) of composable morphisms in \( \mathcal{C} \),

\[ \gamma_{f,g} : f^\alpha \circ g^\alpha \to (f \circ g)^\alpha \]

an isomorphism \( \gamma_{f,g} \) of functors:

\[ \gamma_{f,g} : (F_Y)^\alpha \to (F_X)^\alpha \]

These data must satisfy the following conditions:

a) \( (\text{id})^\alpha = \text{id} \)

b) \( \gamma_{f,f^{-1}} = \text{id} \)

for every three morphisms \( f, g, h \) in \( \mathcal{C} \):

\[ x \xrightarrow{f} y \xrightarrow{g} z \xrightarrow{h} t \]

the following diagram is commutative:

\[
\begin{array}{ccc}
((hg)f)^\alpha & \xrightarrow{\gamma_{hg,f}} & (gf)^\alpha h^\alpha \\
\downarrow^{c_{f,h,g}} & & \downarrow^{c_{f,g,h}^\alpha} \\
F_{f,h,g} & & (f \circ (g^{-1}h)^\alpha) = (f^\alpha (g^{-1}h)^\alpha)
\end{array}
\]

Dualising (taking the category \( \mathcal{C}^0 \) instead of the category \( \mathcal{C} \))

one obtains the notion of **cofibred category**.

### 1.1. Examples

1. Let \( \mathcal{C} \) be the category of ringed spaces and for