TOWARDS AN EXPERT SYSTEM IN STOCHASTIC CONTROL:
OPTIMIZATION IN THE CLASS OF LOCAL FEEDBACKS

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I INTRODUCTION

Stochastic control problems can be solved completely or approximatively by different kind of approaches:

- dynamic programming
- decoupling technique
- stochastic gradient
- perturbation method.

The set of these methods are described in THEOSYS [11] for example.

For each approach we are designing a generator of program able to write automatically fortran program solving the problem.

In Gomez-Quadrat-Sulem [10] we have described a set of automatic tools to solve the problem by the dynamic programming approach.
In this paper we explain the decoupling approach, discuss the possibility of the corresponding generator. Then we give an example of generated program and the numerical results obtained by this generated program.

The plan is the following:

I. INTRODUCTION

II. OPTIMIZATION IN THE CLASS OF LOCAL FEEDBACKS

III. THE GENERATOR OF PROGRAM

IV. EXAMPLE

We want solve the stochastic control problem for diffusion processes that is

\[
\min_u \mathbb{E} \int_0^T C(t,X_t,U_t)dt
\]

where \( U_t \) is the control and \( X_t \) is a diffusion process satisfying the stochastic differential equation

\[
dX_t = b(t,X_t,U_t)dt + \sigma(t,X_t)dW_t
\]

where \( W_t \) denotes a brownian motion \( b \) and \( \sigma \) are given functions.

When \( X_t \) belongs to \( \mathbb{R}^n \) large perhaps larger than 3 or 4 the traditional dynamic programming approach cannot be used practically. We have to apply other methods which do not give the optimal feedback but a good one or the optimum in a subclass of the general feedback class.

In the next paragraph we explain the way of computing the optimal local feedback that is we suppose that each control is associated to a subsystem described by a subset \( I_i \) of the component of \( X_t \) and depends only of the corresponding components of the state.

\[
U : (X_j, j \in I_i) \rightarrow \mathbb{R}
\]

\( U I_i = \{1, \ldots, n\} \) where \( n \) is the dimension of \( X \).