We present an analogue of Clifford's tensor-factorization theorem for principal indecomposable (projective indecomposable) characters, i.e. the Brauer characters afforded by indecomposable projective modules over group algebras over splitting fields of prime characteristic, and discuss some related results.

1. The Statement

The Brauer characters of a finite group $X$ for a prime $p$ are usually defined on the set $X_p$, of $p$-regular elements of $X$. Instead, I prefer to define them as functions on all of $X$ having their usual values on $X_p$, and vanishing on $X - X_p$, (cf. [8]). (This superficial change lets us use the same inner product for Brauer characters as for ordinary characters, and makes the principal indecomposable characters be ordinary characters and Brauer characters at once.) Let $IBr(X)$ denote the set of all irreducible Brauer characters of $X$ in this sense.

If $N$ is a normal subgroup of a finite group $G$, each class-function $\tau$ on $G/N$ vanishing outside $(G/N)_p$, determines a class-function $\inf_p \tau$ on $G$ vanishing outside $G_p$, by

$$\inf_p \tau (g) = \begin{cases} \tau (gN) & \text{if } g \in G_p, \\ 0 & \text{if } g \in G - G_p. \end{cases}$$

The map

$$\psi_j \mapsto \inf_p \psi_j : IBr(G/N) \to IBr(G)$$

is an injection, corresponding to the regarding of representations of $G/N$ as representations of $G$. If $\theta \in IBr(N)$, let $IBr(G|\theta)$ be the
set of those $\phi_j \in \text{IBr}(G)$ whose restrictions $(\phi_j)_N$ to $N$ contain $\theta$ as an irreducible constituent. With this terminology we can state the character-theoretic part of a special case of a theorem of Clifford [2, Theorem 3 and 5] as follows.

Theorem 1 (Clifford). Let $\theta \in \text{IBr}(N)$ for a normal subgroup $N$ of $G$. Assume that $\theta = \xi_N$ for some $\xi \in \text{IBr}(G)$. Then there is a bijection

\[
\psi_j \mapsto \phi_j : \text{IBr}(G/N) \to \text{IBr}(G|\theta)
\]

given by

\[
\phi_j = \xi \inf_p \psi_j.
\]

Observe that the Brauer $1$-character $\psi_1$ of $G/N$ is mapped on $\phi_1 = \xi$.

For any $X$ there is a canonical bijection of $\text{IBr}(X)$ to the set $\text{PI}(X)$ of principal indecomposable characters of $X$. In particular we have bijections

\[
\psi_j \mapsto \psi_j : \text{IBr}(G/N) \to \text{PI}(G/N),
\]

\[
\phi_j \mapsto \phi_j : \text{IBr}(G|\theta) \to \text{PI}(G|\theta),
\]

where (1.6) defines $\text{PI}(G|\theta)$ as a subset of $\text{PI}(G)$. Then Theorem 1 yields a bijection

\[
\psi_j \mapsto \phi_j : \text{PI}(G/N) \to \text{PI}(G|\theta).
\]

It is natural to ask whether (1.7) can be obtained from a formula like (1.4). An answer to this question is given by our main theorem, as follows.

**Theorem 2.** Let $\theta \in \text{IBr}(N)$ for a normal subgroup $N$ of $G$. Assume that $\theta = \xi_N$ for some $\xi \in \text{IBr}(G)$. Then: