EXTENSION OF CR-FUNCTIONS

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Summary. Let \( \Gamma \) be a real hypersurface in \( \mathbb{C}^n \), \( n \geq 2 \), oriented of class \( C^1 \), compact, connected with boundary \( \partial \Gamma \). Suppose that \( \partial \Gamma \) belongs to a \( C^\infty \)-hypersurface \( M \) and there exists a relatively open subset \( A \) of \( M \) such that \( \partial A = \partial \Gamma \), and that \( \Gamma \cap M = \partial \Gamma \). Under these hypotheses, in a previous paper [2] G. Lupaccioli and the author have proved that if \( M \) is the zero-set of a pluriharmonic function on a neighbourhood of \( \bar{D} \), every Lipschitz continuous CR-function \( f \) on \( \Gamma^0 = \Gamma \setminus \partial \Gamma \) extends uniquely by a function \( F \), holomorphic on \( D \), where \( D \) is an open set of \( \mathbb{C}^n \) having \( \Gamma \cup A \) as its boundary. Moreover, \( F \) is continuous on \( D \cup \Gamma^0 \). The present paper aims at generalizing this result to the case when \( M \) is Levi-flat or Levi-pseudoconcave with respect to \( D \). The positive answer is obtained in two particular cases (Theorems 1 and 3).

1. Let \( \Gamma \) be a real hypersurface in \( \mathbb{C}^n \), \( n \geq 2 \), oriented of class \( C^1 \), compact, connected with boundary \( \partial \Gamma \). Assume the following conditions are verified:

I) \( \partial \Gamma \) belongs to a \( C^\infty \)-hypersurface \( M \) and there exists a relatively open subset \( A \) of \( M \) such that \( \partial A = \partial \Gamma \),

II) \( \Gamma \cap M = \partial \Gamma \).

Let \( D \) be the open set of \( \mathbb{C}^n \) having \( \Gamma \cup A \) as its boundary. In [2] the following theorem has been proved:

**Theorem.** Let \( M \) be the zero-set of a pluriharmonic function on a neighbourhood of \( \bar{D} \). Then every Lipschitz continuous CR-function \( f \) on \( \Gamma^0 = \Gamma \setminus \partial \Gamma \) extends, in a unique way, by a function \( F \), holomorphic on \( D \) and continuous on \( D \cup \Gamma^0 \).

In particular, every holomorphic function on \( D \), regular on \( \bar{D} \)
is completely determined by its values on $\Gamma$. Then it is natural to conjecture that such a property is still valid when $M$ is Levi-flat or Levi-pseudoconcave with respect to $D$.

The aim of this paper is to give a positive answer to this conjecture in two particular cases (Theorems 1 and 3).

2. In the first case $\Gamma$ and $M$ verify the following conditions:

a) $\Gamma$ is contained in the boundary of an open subset $\Omega \subset \subset \mathbb{R}^n$ defined by a $C^\infty$-function $\phi$, plurisubharmonic on a neighbourhood $U$ of $\overline{\Omega}$;

b) $M$ is the zero-set of a function $\rho \in C^\infty(U)$ and $\Gamma \subset \{z \in U : \rho(z) \geq 0\}$; moreover, on $\{z \in U : \rho(z) > 0\}$, $\rho$ is plurisubharmonic and strictly plurisubharmonic if $n = 2$.

**Theorem 1.** Every Lipschitz continuous CR-function $f$ on $\Gamma^0$ extends by $\xi$-function $F$, holomorphic on $D$ and continuous on $D \cup \Gamma^0$.

We make some remarks before going into the proof. First, we may assume that $U$ is a Stein domain, $\phi$ is plurisubharmonic on $U$, and, moreover, that $f$ is continuous on a neighbourhood of $\Gamma$. For $\delta > 0$ we set $\Omega^+ = \{z \in U : \phi(z) < \delta\}$ and $\Omega^0 = \{z \in \Omega^0 : \rho(z) > 0\}$ in such a way that $\Omega^0 = \{z \in \Omega^0 : \rho(z) > 0\}$ is a Stein domain and $0(\Omega^0)$ is a dense subspace of $0(\Omega^0)$; [3]. For fixed $\epsilon_0 > 0$ let $A^* = A_{\epsilon_0}$ be the intersection of $\Omega^0$ with the hypersurface $\rho^* \equiv \rho_0 = 0$ and let $A^*$ be the $0(\Omega^0)$-envelope of $A^*$. $A^*$ is contained in $\{z \in \Omega^0 : \rho^*(z) \geq 0\}$ as follows from the fact that $0(\Omega^0)$ is dense in $0(\Omega^0)$ for every $\epsilon > 0$.

Let $D(\epsilon_0) = \{z \in D : \rho^* (z) > 0\}$ and for $g \in 0(\Omega^0)$ let $W_g = \{z \in \Omega^0 : |g(z)| > \lVert g \rVert_{\Omega^0}\}$; $D$ is contained in $\bigcup W_g$.

**Lemma.** Let $\zeta \in \Omega^0$ be such that $\rho^*(\zeta) > 0$. Then there exist $g \in 0(\Omega^0)$ and a connected subset $C$ verifying the following properties:

1) $C \subset \{z \in \Omega^0 : \rho^*(z) > 0\}$ and $C \cap D(\epsilon_0)$, $C \cap (\Omega^0 \setminus \Omega^0)$ are connected;

2) $C \cap \partial \Omega^0 \neq \emptyset$ and $C \subset W_g$, the connected component of $W_g$ containing $\zeta$;

3) $C \cap \{z \in \Omega^0 : |g(z)| > \lVert g \rVert_{D(\epsilon_0)}\} \neq \emptyset$.

**Proof.** As $\zeta \in A^*$, there is a smooth, connected complex hypersurface $Z$ of a neighbourhood of $\Omega^0$ containing $\zeta$ and such that