Chapter 1

Generalized Reduced Gradient and Global Newton Methods

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1. INTRODUCTION

The object of this paper is to show how to solve a system of n nonlinear equations

\[ f(x) = 0, \quad f : \mathbb{R}^n \to \mathbb{R}^n \]

by the Global Newton (GN) method, using the General Reduced Gradient (GRG) method as a numerical tool. The method thus obtained is applied to the general nonlinear programming problem with equality or inequality constraints. More than one local optimum may be obtained by the method.

We first briefly review our notations and some algebraic prerequisites (Section 2). Section 3 reviews some features of the GN method. We show in Section 4 how the GRG method is applicable to GN, then we briefly explain in Section 5 how the method of Section 4 may be used for nonlinear programming problems. We present some numerical experiments in Section 6.

2. NOTATIONS AND ALGEBRAIC PRELIMINARIES

\( x \) is any point in \( \mathbb{R}^n \), identified with its column-matrix of compo-
nents $x_i$, $i = 1, \ldots, n$. $f$ is a mapping $\mathbb{R}^n \to \mathbb{R}^n$, $f \in C^2[\mathbb{R}^n]$. $f(x)$ is identified with its column-matrix of components $f_i(x)$, $i = 1, \ldots, n$. $f'(x)$ is the derivative of $f(x)$, identified with the $(n,n)$ matrix whose elements are $\frac{\partial f_i}{\partial x_j}$, the row-indices $i$ and the column-indices $j$ running from 1 to $n$. The matrix $f'(x)$ may be written row-wise as

$$f'(x) = \begin{pmatrix} f'_1(x) \\ \vdots \\ f'_n(x) \end{pmatrix}$$

where

$$f'_i(x) = \left( \frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n} \right),$$

or column-wise

$$f'(x) = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix},$$

where

$$\frac{\partial f}{\partial x_j} = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$$

We shall need the adjoint matrix $f'(x)^a$ of $f'(x)$, defined by its elements

$$(f'(x)^a)_j,i = (-1)^{i+j} \det \left\{ (f'(x) \backslash f'_i(x)) \backslash \frac{\partial f}{\partial x_j} \right\},$$

where the symbol $(\backslash)$ means "remove", so that the right hand side is the co-factor of $\frac{\partial f_i}{\partial x_j}$. We then have the relation

$$f'(x)f'(x)^a = f'(x)^a f'(x) = \det(f'(x))I_{n,n},$$

where $I_{n,n}$ is the $(n,n)$ identity matrix.

$x^0$ is a particular point in $\mathbb{R}^n$, which may have different meanings in different Sections.

We now recall, for completeness, some prerequisite from linear al-