INDECOMPOSABLE REPRESENTATIONS
OF FINITE ORDERED SETS

Michèle Loupias

In this paper I shall denote a finite (partially) ordered set, \( k \) a (commutative) field and \( \mathcal{E} \) the category of the finite dimensional vector spaces over \( k \). If we consider \( I \) as a category we note \( \mathcal{I} = \text{Hom}(I, \mathcal{E}) = \) the category of functors from \( I \) to \( \mathcal{E} \). An object \( E \) of \( \mathcal{I} \) is called a representation of \( I \). The category \( \mathcal{I} \) is a Krull-Remak-Schmidt category.

The set \( I \) is said of finite representation type (F.R.T) if \( \mathcal{I} \) has only a finite number of indecomposable objects (up to isomorphism). The purpose of this work is to determine all the sets of F.R.T. We shall suppose from hereon that \( I \) is connected.

1. GENERALITIES - CRUCIAL SETS - CRITICAL SETS

Some well known sets of F.R.T are the sets \( A_n, D_n, E_6, E_7, E_8 \) defined in [2] and the cycles defined in [1].

1.1- PROPOSITION - The following sets are not of F.R.T

\[ E_6 = c_2 - c_1 - a - d_1 - d_2 \]
\[ E_7 = c_3 - c_2 - c_1 - a - d_1 - d_2 - d_3 \]
\[ E_8 = c_2 - c_1 - a - d_1 - d_2 - d_3 - d_4 - d_5 \]
\[ D_1 = b_4 - a - b_2 \]
\[ A_4 = a \]
\[ R_1 = a \]

\[ b \]
\[ c \]
\[ d \]
\[ b_1 \]
\[ b_1 \]
\[ b_1 \]
The result is known for the sets $E_6$, $E_7$, $E_8$, $D_4$ and $A_4$.

For $R_1$, $R_2$, $R_3$, $R_4$ it follows from an equivalence between the category of the representations $E$ of the set $a - a_1$ such that $\ker E(c - a) \cap \ker E(c - b) = 0$ and the category of the representations $F$ of the set $a' \to s' \to b'$ such that $\text{Im } F(a' - s') + \text{Im } F(b' - s') = F(s')$: to $E$ we associate $F$, by setting $F(s') = \text{the fiber coproduct of } E(a)$ and $E(b)$ under $E(c)$, and $F(a') = E(a)$, $F(b') = E(b)$, $F(d') = E(d)$.

Let $\mathcal{A}$ be the category of filtered vector spaces $A$, with the filtration $B' \to X \to A \leftarrow A_4 \leftarrow A_3 \leftarrow A_2 \leftarrow A_1 \leftarrow A \leftarrow C \to Y$.