Let $k$ be a (commutative) field, and $k\langle X_1, \ldots, X_n \rangle$ the free associative algebra in $n$ (non-commuting) variables. Denote by $M_i$ the ideal of $k\langle X_1, \ldots, X_n \rangle$ generated by all monomials of degree $i$. For any $k$-algebra $A$, let $\tilde{A}$ be the category of all $A$-modules which are finite dimensional as $k$-vector spaces. If $I$ is a twosided ideal of $k\langle X_1, \ldots, X_n \rangle$, then for $A = k\langle X_1, \ldots, X_n \rangle/I$, the category $\tilde{A}$ is just the category of all (finite dimensional) vector spaces endowed with $n$ endomorphisms which satisfy the relations expressed by the elements of $I$.

The $k$-algebra $A$ is called **local**, provided $A = k \cdot 1 + \operatorname{rad} A$, where $\operatorname{rad} A$ is the Jacobson radical of $A$. If $A$ is a local $k$-algebra, we will consider also its completion $\overline{A} = \lim_{\leftarrow} A/(\operatorname{rad} A)^n$. There is a canonical ring homomorphism $A \rightarrow \overline{A}$, and $A$ is said to be **complete** in case this homomorphism is an isomorphism. Since obviously every object in $\tilde{A}$ is annihilated by some power $(\operatorname{rad} A)^n$, the canonical homomorphism $A \rightarrow \overline{A}$ induces an isomorphism of the categories $\tilde{A}$ and $\overline{A}$. Thus, in order to consider the behaviour of $\tilde{A}$ for a local algebras $A$, we may restrict to the case where $A$ is complete.

The $k$-algebra $A$ is said to be **wild** (or to be of wild representation type) provided there is a full and
exact subcategory of $\mathcal{A}_m$ which is representation equivalent to the category $k\langle X, Y, Z \rangle$. The reason for calling it wild, is that there seems to be no hope to expect a complete classification of the indecomposable objects in $k\langle X, Y, Z \rangle$, since for any finitely generated $k$-algebra $B$, there is a full and exact embedding of $B_m$ into $k\langle X, Y, Z \rangle$. On the other hand, the algebra $A$ is said to be tame (or to be of tame representation type), if there exists a complete classification of the indecomposable objects in $A_m$, and if there are not only finitely many indecomposables.

In order to distinguish the complete local algebras according to their representation type, we have to find the smallest possible wild algebras (that is, wild algebras for which all proper residue algebras are tame or of finite representation type), and the largest possible tame algebras (that is, tame algebras which do not occur as proper residue algebras of other tame algebras).

(1.1) We will have to consider several algebras which we want to introduce now. First, we mention
(a) $k\langle X, Y, Z \rangle/M_2$, the local algebra of dimension 4 with radical square zero. Next, we single out certain residue algebras $k\langle X, Y \rangle/I$ of $k\langle X, Y \rangle$ of dimension 5, namely those with $I$ the twosided ideal generated by the elements
(b) $X^2, XY, X^2Y, Y^3$;
(b') $X^2, XY, XY^2, Y^3$;
(c) $X^2, XY - \alpha YX, X^2Y, Y^3$ with $\alpha \neq 0$; and
(d) $X^2 - Y^2, YX$. 