quasi-hyperbolic orbit as in cases (1) and (2) defined earlier. In particular, are the arcs joining an Anosov diffeomorphism of $T^2$ to a DA (derived from Anosov) structurally stable? Another specially interesting question is for arcs joining two Morse-Smale diffeomorphisms. If $\xi_b$ has a quasi-hyperbolic orbit as in (3) or a quasi-transversal intersection as in (4), then the arc is not structurally stable.

References.
3. J. Palis & C. Pugh, 50 problems in dynamical systems, these Proceedings.

For motivation to possible applications see:


Let $M$ be a compact $C^\infty$ manifold without boundary and denote by $C^k(I, X^r(M))$ the space of $C^k$ curves of $C^r$ vector fields with the $C^k$ topology $k \geq 1$, $r \geq 2$. Let $AS$ be the subspace of vector fields satisfying Axiom A and strong transversality. Define the bifurcation set of $\xi \in C^k(I, X^r(M))$ to be $B(\xi) = \xi^{-1}(X^r(M) - AS)$. We say $\xi$ is simple if it has a neighbourhood $\mathcal{U}$ in $C^k(I, X^r(M))$ such that, for any, $\eta \in \mathcal{U}$, card $B(\xi) = \text{card } B(\eta) < \infty$. If there is a simple arc between $X$ and $Y$ we say $X$ and $Y$ are simply related, and we write $X \equiv Y$.

Let $\mathcal{F}^k = C^k(M, R)$ be the space of $C^k$ real-valued functions on $M$ and let $\mathcal{G}^k$ be the space of $C^k$ Riemannian metrics on $M$. Theorems 1 and 2 below are joint work with M. Peixoto [2] while theorems 3 and 4 are later extensions by the author [1].
For \( f \in \mathcal{C}^k \), \( g \in \mathcal{C}^{k-1} \), define \( \text{grad} f \) by \( g_x(\text{grad} f(x), y) = df_x(y) \) for \( x \in M, \ Y \in T_xM \).

**Theorem 1.**
(a) Fix a \( C^\infty \) Morse function \( f:M \to \mathbb{R} \) (i.e. a function having only non-degenerate critical points). For a dense open set of arcs \( g_t \) in \( \mathcal{C}^k(I, \mathcal{C}^r) \), \( k \geq 1, r \geq 2 \), the curve \( t \to \text{grad} g_t f \) is simple.

(b) Fix a \( C^\infty \) Riemannian metric \( g \) on \( M \). For dense open set of arcs in \( \mathcal{C}^k(I, \mathcal{C}^r) \), \( k \geq 1, r \geq 3 \), the curve \( t \to \text{grad} g_t f \) is simple.

The proof of theorem 1 uses transversality theory and refinements of the techniques of Smale [4] and Palis [3].

**Theorem 2.** Any two Morse-Smale vector fields are simply related.

We indicate the proof of theorem 2. Let \( MS \) be the set of Morse-Smale vector fields on \( M \) and \( X \in MS \). First one breaks the closed orbits of \( X \) with careful creations of saddle nodes (see [1] for a more refined use of this technique). Thus \( X \equiv X_1 \) where \( X_1 \in MS \) and is gradient-like. Then \( X_1 \equiv X_2 \) with \( X_2 \in MS \) and \( X_2 \) locally a gradient. By Smale [4] there is a Morse function \( f \) and a Riemannian metric \( g \) with \( X_2 = \text{grad} g f \). Similarly if \( Y \in MS \) we find \( f', g' \) s.t. \( Y \equiv \text{grad} g f' \). Changing \( f' \) slightly, if necessary, we may assume \( \text{grad} g f' \in MS \). Choose curves \( \{G_t\} \in \mathcal{C}^k(I, \mathcal{C}^r) \) and \( \{F_t\} \in \mathcal{C}^k(I, \mathcal{C}^r) \) with \( G_0 = g, G_1 = g', F_0 = f, F_1 = f' \). Theorem 1(a) gives \( Y \equiv \text{grad} g f' \) and theorem 1(b) gives \( X \equiv \text{grad} g f \). Hence \( X \equiv Y \).

**Theorem 3.** Any AS vector fields \( X, Y \) whose non-wandering sets are \( d \)-separated (see [1] for definition) are simply related.

**Theorem 4.** Any AS vector fields \( X \) and \( Y \) on \( M \) are simply related provided \( \dim M < 4 \).

The proof of theorem 3 proceeds as in theorem 2 after first breaking up the non-trivial basic sets via saddle-nodes. We do not know whether any two AS vector fields on \( M \) must be simply related if \( \dim M \geq 4 \).

**Remark.** There do not seem to be any general results for diffeomorphisms similar to those above for flows. (The rotation number on the invariant circle formed in the Hopf bifurcation prevents us from approximating this bifurcation by a simple arc of diffeomorphisms.) If \( \overline{A}:\mathbb{T}^n \to \mathbb{T}^n \) is an Anosov diffeomorphism of the \( n \)-torus not of codimension one we have been unable to find a simple arc between \( \overline{A} \) and any topologically different AS diffeomorphism.