SOME THOUGHTS ON THE FUTURE OF CATEGORY THEORY

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The Como meeting was something of a milestone, coming as it did just twenty-five years after the first international meeting on category theory held at La Jolla, California in 1965. The work of Kan, Grothendieck, and others had greatly intensified the elaboration and application of the subject in the ten years prior to La Jolla, and enormous development has continued uninterruptedly since. I have been asked, as a participant at both meetings, to speculate on how at least some of the threads of the subject might develop in the immediate future. The threads I have selected now were only dimly visible then, for when J. L. Verdier described topos theory on the beach at La Jolla, most of us were slow to grasp its significance.

The crystallized philosophical discoveries which still propel our subject include the idea that a category of objects of thought is not specified until one has specified the category of maps which transform these objects into one another and by means of which they can be compared and distinguished. Thus, for applications of mathematics, to objectify is to mapify. Quite non-trivial in fact is also the idea that there must be definite domains and definite codomains and that there must be identity maps; even today there are many who think one could usefully "generalize" by omitting those requirements, sometimes on grounds of dislike for the "stasis" they think they imply. However, in modern Greek "stasis" means "bus-stop"; how useless an intricate network of speeding buses would be without them, and how disembodied would be processes without states. In fact category theory is the first to capture in reproducible form an incessant contradiction in mathematical practice: we must, more than in any other science, hold a given object quite precisely in order to construct, calculate, and deduce; yet we must also constantly transform it into other objects. These precepts, together with the powerful guide to look for and use adjoints in all categories large and small because they are the form of most constructions and deductions and many calculations and estimates, have guided us in our work in all

the varied fields of mathematics. Most of us have struggled to explicitly introduce these principles also into our teaching, and those who have persisted find that this explicit use of the unity and cohesiveness of mathematics sparks the many particular processes whereby ignorance becomes knowledge, in learning just as it does in investigating. The need to teach, to explain and to respond to students' probing is often the genesis of problems taken up in "pure" research.

Though much remains to be done, it seems to some that we (that is, the community of category theorists with our ties to all the fields of pure and applied mathematics) have reached a unique position with regard to philosophy. I concentrate here on an outline of what is intended as a positive mathematical program. The history of possible philosophical objections to it will be treated elsewhere. Suffice it to suggest that Möbius, Hamilton, Grassman, Maxwell, etc. would not be among the naysayers. At least we can hope that sober application, of category theory to the ancient philosophical categories, will not only clarify both but also renew respect for serious thought, through solid examples approaching adequacy to their concept.

This attempt, by an admirer of rational mechanics, to include objective logic among the tools for arriving at a more accurate conception of space, will, I hope, not be dismissed by confusing it with objective idealism. The general science of the development of scientific ideas has a big overlap with category theory. That general science does not claim that scientific ideas are self-generating nor does it depend on faith for the acceptance of its own conclusions, as idealism would.

In the first section I start from the opposition between connected and separable objects to propose the tentative clarification, by a certain disjoint pair of classes of categories, of the conceptions of Being and Becoming respectively; how the one class arises from the other is the content of some resulting mathematical conjectures. In the second section, a specific mathematical formulation of the principle "unity-and-identity-of-opposites" is described in hopes of clarifying dimensionality in general and infinitesimals in particular, with again some mathematical conjectures aiming at further clarification. In the third section it is urged that certain pathologies "commonplace" since 1861/1890 need not be included in a more accurate conception of space and that both more physically-realistic models of computers as well as a more "objective" approach to Diophantine problems are already emerging from certain fascinating calculations.