Many problems for partial differential equations can be represented as problems for evolutionary equations in the corresponding Banach spaces \([1,2]\). The applications of the geometric theory of differential equations to the investigation of differential equations in Banach spaces is traditional \([2,3]\). The integral manifolds method is part of this \([4]\).

In this paper the integral manifolds method is applied to the investigation of abstract parabolic systems of differential equations

\[
\frac{dx}{dt} = A(t)x + f(t,x,y), \quad \frac{dy}{dt} = B(t)y + g(t,x,y)
\]

where \(x\) and \(y\) are elements of Banach space \(X\) and \(Y\) with norms \(\| \cdot \|\), \(t \in \mathbb{R}\).

Let us suppose that for (I) the following conditions are satisfied.

I. Nonlinear operator functions \(f: \mathbb{R} \times X \times Y \rightarrow X\) and \(g: \mathbb{R} \times X \times Y \rightarrow Y\) and all their partial derivatives to second order are continuous and bounded on the set \(\mathbb{R} \times X \times Y (\varrho)\) where \(B(\varrho) = \{ y | \| y \| \leq \varrho \}\).

II. For every \(t\) the unbounded operator \(B(t)\) is linear and closed and it has dense in \(Y\) independent with respect to \(t\) domain of definition \(D(B(t)) = D(B)\). \(B(t)\) is strongly continuous on \(D(B)\) and \(\| (A - \lambda I)^{-1} \| \leq I\) when \(\lambda > 0\). The evolutional operator \(U(t,s)\) of the homogeneous equation

\[
\frac{dy}{dt} = B(t)y
\]

satisfies the inequality...
The evolutional operator $V(t,s)$ of the homogeneous equation
\[ \frac{dx}{dt} = A(t)x \]
for linear continuous bounded operator $A(t)$ satisfies the inequality
\[ \| V(t,s) \| \leq Ke^{-\beta(t-s)} \]
(2)
\[ K \geq 1, \beta > 0, \quad t \geq s. \]

The boundedness of $f$ and $g$ and their first partial derivatives implies the existence of positive numbers $\alpha$, $\lambda$, and $c$ such that for all $t \in \mathbb{R}$, $x, \bar{x} \in X$, $y, \bar{y} \in B(\mathcal{P})$ the following inequalities hold
\[ \| f(t,x,y) \| \leq a, \quad \| g(t,x,y) \| \leq a; \]
(4)
\[ \| f(t,x,y)-f(t,\bar{x},\bar{y}) \| \leq \lambda \| x-\bar{x} \| + c \| y-\bar{y} \| ; \]
\[ \| g(t,x,y)-g(t,\bar{x},\bar{y}) \| \leq \lambda (\| x-\bar{x} \| + \| y-\bar{y} \| ) . \]

Henceforth it is assumed that the number $\lambda$ is sufficiently small.

Using the standard scheme [2-4], it is easy to establish the existence of nonlocal integral manifold $y = h(t,x)$, $t \in \mathbb{R}, x \in X$ of system (I). The function $h$ is found in the complete metric space $M$, whose elements are continuous functions $\mathbb{R} \times X \rightarrow B(\mathcal{P})$ which satisfy the inequalities
\[ \| h(t,x) \| \leq D, \quad \| h(t,x)-h(t,\bar{x}) \| \leq \lambda \Delta \| x-\bar{x} \| \]
(5)
for all $t \in \mathbb{R}$ and $x, \bar{x} \in X$. The metric on $M$ is defined by
\[ d(h,\bar{h}) = \sup_{\mathbb{R} \times X} \| h(t,x) - \bar{h}(t,x) \| . \]

The function $h$ satisfies the operator equation