Invariant parabolic equations of the second order naturally connected with phase space geometry appear in various problems of global analysis, theoretical and mathematical physics. [1 ].

Stochastic process theory may suggest a method for solving those equations which is weakly affected by the phase space dimension growth. Besides this method allows to investigate equations with degenerate coefficients.

Invariant parabolic equations on infinite dimensional manifolds and vector bundles had been investigated in [2 - 4]. Here we extend the results of [3 ] to adjust them to the additional structures on manifolds such as principal fibre bundle and associated vector bundle total space structures. On this way we succeed in studying the Cauchy problem both for the equation

\[ \frac{\partial \mathcal{C}}{\partial s} + \frac{1}{2} \text{Tr}(\nabla \mathcal{F}_{Q_z(s)} \cdot \nabla \mathcal{F}_{Q_z(s)} - \nabla \mathcal{F}_{Q_z(s)} \cdot \mathcal{F}_{Q_z(s)} + \nabla \mathcal{F}_{Q_z(s)} \mathcal{F}) = 0 \]

with respect to a scalar function \( \mathcal{C}(t,z) \) defined on principal fibre bundle total space \( \mathcal{P} \) and for the equation

\[ \frac{\partial f}{\partial s} + \frac{1}{2} \text{Tr} \left\{ \nabla \text{Ad}^*(p) \cdot \nabla A_x(s) + \nabla \text{Ad}^*(p) \cdot \mathcal{F} - \nabla \text{Ad}^*(p) A_x(s) \cdot \mathcal{F} \right\} + \text{Tr} \mathcal{C}_x(s) \cdot \mathcal{F} - \mathcal{F} = 0 \]
with respect to the section $s_A$ of the associated vector bundle $\text{Ad}^*(p)$.

The first section of the work is devoted to a description of various differential geometry objects needed for a strict formulation of the problems in question and those objects which have to be used due to the chosen way of investigation. In the second section we describe and study invariant stochastic equations in principal and associated bundles while the last section deals with probabilistic representation of the above-mentioned Cauchy problem solutions.

In this section we also state some conditions to ensure the problems in question to be correct in a classical sense.

Let $X$ be a smooth Banach manifold modelled on a separable Banach space $B$ endowed with a smooth norm. Denote by $p: \mathcal{P} \rightarrow X$ a principal fibre bundle with the structural group $G$ acting on $\mathcal{P}$ from the right. Suppose $G$ is a Lie group and $\mathfrak{g}$ is its Lie algebra. Since $\mathcal{P}$ is locally trivial then for any chart of the atlas $(U_\alpha, \varphi_\alpha)$ of the manifold $X$ there exists a diffeomorphism $p^{-1}(U_\alpha) \rightarrow U_\alpha \times G$ such that $z \mapsto (p(z), S_\alpha(z))$. For each $x \in U_\alpha \cap U_\beta$ we may define $S_{\alpha, \beta}(z) = S_\alpha(z) S^{-1}_{\beta}(z)$ and note that in fact $S_{\alpha, \beta}$ depends on $p(z) = x$ only due to the relation

$$S_\alpha(z) S^{-1}_{\beta}(z) = S_\alpha(z) a^{-1} S^{-1}_{\beta}(z) = S_\alpha(z) S^{-1}_{\beta}(z)$$

At last one can see that the map $z: G \rightarrow \mathcal{P}$ given by $z(g) = R_g z$ defines a special diffeomorphism of $G$ into a fibre at the point $p(z)$.

To describe a vector bundle $V : E \rightarrow X$ associated with the principal fibre bundle $p: \mathcal{P} \rightarrow X$ and such that it has a Banach space $F$ for its fibre on which $G$ acts from the left (possibly by means of a linear representation) we use the following construction. Let $Q = \mathcal{P} \times F$, consider the right action of $G$ on $Q$ $(z, f) g = (z g, g^{-1} f)$ with $z \in \mathcal{P}$, $f \in F$, $g \in G$. Denote by $\mathcal{E} = \mathcal{Q} / G$ a set of $G$-orbits of $Q$-points. This set is taken for a total space of the associated vector bundle. The projection map $\pi: \mathcal{E} \rightarrow X$ is given by the relation $\pi((z, f) G) = p(z)$. The map $S_u: p^{-1}(U) \rightarrow G$ gives rise to a map $\mathcal{P}_u : \pi^{-1}(U) \rightarrow F$ such that $\mathcal{P}_u((z, f) G) = S_u(z)f$. Assuming that this map is a diffeomorphism we endow $\mathcal{E}$ with a structure of a smooth vector