§1 Introduction

The following classic problem is of fundamental interest in mathematical physics.

Determine a scalar valued function \( U(r;k) \) such that

\[
\begin{align*}
(i) & \quad (\Delta + k^2)U = f(r) \quad \text{in an infinite domain } D \text{ with boundary } \partial D, \\
(ii) & \quad \alpha U + \beta \frac{\partial U}{\partial n} = g(r), \quad r \in \partial D \\
\end{align*}
\]

where \( \alpha, \beta \) are given constants, \( g(r) \) is prescribed on \( \partial D \) and \( \frac{\partial}{\partial n} \) is the outward directed (in to \( D \)) normal derivative to \( \partial D \).

\[ U(r;k) = U^i(r;k) + U^s(r;k) \quad \text{where } U^i(r;k) \text{ is the prescribed incident field, } U^s(r;k) \text{ is the scattered field which satisfies } (\Delta + k^2)U^s = 0 \]

\[ U^s(r;k) \text{ satisfies the Sommerfeld radiation condition, e.g. } \lim_{|r| \to \infty} \frac{|r|^{\frac{3}{2}} e^{-ikt}}{|r|} = 0 \]

where \( t \) is chosen depending on the assumed harmonic time dependence which has been suppressed.

Exact solutions to such problems are known only in certain special cases. For example the so-called canonical problems wherein \( \partial D \) is a normal surface in a particular orthogonal curvilinear coordinate system. In this case the problem may be solved using the classic separation of variables technique.

In general exact solutions cannot be obtained and even for canonical problems the solutions are often too intractible for useful application. Thus there is considerable interest in approximate methods of solution. If the wave number \( k \) is large (\( k = 2\pi/\lambda, \lambda = \text{wave length} \)) the modern geometrical theories of optics and diffraction provide what are conjectured to be asymptotic approximations of exact solutions in the illuminated and shadow portions of \( D \).
For the problem of scattering by an arbitrary convex cylinder of finite cross section it has been established that the field predicted by the geometrical theory of optics is an asymptotic solution in the illuminated region. This was established for the Neumann problem (set $\alpha = 0$ in (1.1) ii) by Ursell [12] in his now classic paper of 1957. Morawetz and Ludwig [9] proved the corresponding result for the Dirichlet problem (set $\beta = 0$ in (1.1) ii). In addition they established the validity of the geometrical theory of optics for the case of a point source radiating outside a closed convex surface on which the Dirichlet condition is imposed. As far as the geometrical theory of optics is concerned these appear to be the most general results known.

The geometrical theory of diffraction however has been verified in only a few special situations. To cite the most complete confirmation we mention Ursell [12] who verified the theory for the case of scattering by a circular cylinder and Leppington [6], Ursell [13] who verified the theory for the case of scattering by an elliptic cylinder.

The first significant attack on establishing the validity of the geometrical theory of diffraction for non canonical problems, that is where the method of separation of variables cannot be applied, is that of Bloom and Matkowsky [1]. In that work they considered the scattering of a wave from an infinite line source by an infinitely long cylinder $C$ say. The line source is parallel to the axis of $C$ and the cross section $\Omega$ of the cylinder is smooth, closed and convex. Furthermore $\Omega$ is formed by joining a pair of smooth convex arcs to a circle $\Omega_{0}$, one on the illuminated side, and one on the dark side, so that $\Omega$ is circular near the points of diffraction. By a rigorous argument Bloom and Matkowsky derived the asymptotic behaviour of the field at high frequencies in the "deep" shadow of $\Omega$. The leading term of the asymptotic expansion was shown to be the field predicted by the geometrical theory of diffraction.

In this paper we describe a genuine extension to the results of Bloom and Matkowsky. The extension is based on the fundamental observation that given an arbitrary closed convex curve, then for any two points on this curve we can