FROST PROPAGATION IN WET POROUS MEDIA

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Résumé : Un milieu poreux saturé d'eau gèle lorsqu'il est soumis à l'action du froid. Le front de gel qui sépare la partie non gelée de la partie gelée est une surface libre. L'expérience montre qu'il apparaît une dépression sur le front de gel. L'eau est alors aspirée vers ce front et gèle en l'atteignant.

Le problème est un problème de Stefan couplé liant les équations de la diffusion de la chaleur et de l'eau. L'équation de la conservation de l'énergie couple les équations sur le front de gel. Les équations d'évolution obtenues sont résolues en introduisant une nouvelle inconnue, l'indice de gel, et en utilisant les techniques des inéquations variationnelles. On présente enfin un exemple numérique.

Summary : A water saturated porous medium freezes when it is chilled. The frost line which separates the frozen part and the unfrozen part is a free surface. Experiments show that a depression appears on the frost line. Water is thus sucked in through the unfrozen part. It freezes when it reaches the frost line.

The problem is a coupled Stefan problem linking the heat and water equations of diffusion. The energy conservation law couples the equations on the frost line. The equations are solved using a new unknown the freezing index and the methods of variationnal inequalities. A numerical example is given.

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I. INTRODUCTION THE PHYSICAL PROBLEM.

A water saturated porous medium freezes when it is chilled. It occupies an open part \( \Omega \) of \( \mathbb{R}^n \) (\( n = 1, 2 \) or 3). The frost line which separates the frozen and unfrozen parts is a free surface: i.e., it is an a priori unknown surface. Experiences show that a depression appears on the frost line. Water is thus sucked in through the unfrozen part. This sucked in water freezes when it reaches the frost line. This accumulation of ice induced by the frost results in an heaving of the structure.

This phenomenon is important for road maintenance in cold weather. It is known that ice accumulation and frost heaving results in a decrease of bearing capacity during thaw.

The physical experiments have allowed the comprehension of the phenomenon and the construction of a mathematical model [1]. The unknowns of the problem are the temperature \( \theta(x,t) \) and the head of water \( h(x,t) \) at any point \( x \) of \( \Omega \) and any time \( t \) of the \([0,T]\) period during which this phenomenon is being investigated. The data are the initial state of the medium and the external actions which determine the boundary conditions. The hydraulic and thermal phenomenons described by the classical diffusion equations are, as already said, coupled. The coupling occurs on the frost line according to the energy conservation law.

This problem is solved in two steps. First we introduce a new unknown: the freezing index, roughly a heat, which is important from the technical and physical point of view. If there is no hydraulic phenomenon this step solves again the thermal problem in a different manner, which is however equivalent to previous results [3, 4, 5, 6, 9, 11, 12, 15]. The advantage of this presentation is to use the freezing index and a formulation in terms of variational inequalities. The second step allows the calculation of the head of water. The problem is entirely solved by the knowledge of the freezing index and of the head of water allowing to compute the frost heaving.

Computer programs are used by the Laboratoire Central des Ponts et Chausées to study the freezing of pavements and to protect them against its harmful consequences.

The equations are established in §II where the freezing index is introduced. The variational formulation is also given in §II. The main results are given in §III: they concern the existence and the uniqueness of solutions. The §IV and V contain the proof pattern. Numerical results are presented in §VI.

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