The purpose of this lecture is to review, for a mathematical audience, some of the connections recently explored between non-equilibrium statistical mechanics and several techniques in functional analysis. In doing so, we hope to stimulate further enquiries into some of the mathematical structures suggested by the process of building our physical understanding on reliable mathematical foundations.

In line with a time-honored tradition in applied mathematics, the first section is a heuristic presentation of the physical question to be addressed. In the second section we use the Nagy dilation technique for semi-groups of contractive operators to extract some positive information from a no-go theorem. We show in the third section the relevance of the concept of spectral concentration for the decay problem. In the fourth section we direct our attention to some analogy between the theory of the asymptotic behavior of solutions of certain stochastic differential equations and the van Hove limit in non-equilibrium statistical mechanics. In the fifth section we turn to certain aspects of the theory of completely positive maps on von Neumann algebras to propose a definition of dynamical systems which extends the classical notions to the quantum realm; as an illustration we discuss the Bloch equation governing spin-relaxation. Finally in the sixth section we briefly indicate some recent advances in the non-commutative extensions of classical ergodic theory.
1. HEURISTICS

The fundamental problem of non-equilibrium statistical mechanics is to reconcile two physical descriptions of nature: non-equilibrium thermodynamics and mechanics.

The thermodynamical description is a macroscopic, phenomenological theory epitomized by dissipative transport equations. An important class of such equations present themselves as parabolic differential equations with time-independent coefficients, the so-called "transport coefficient". A typical example is provided by the diffusion equation:

\[ \frac{\partial \rho(x,t)}{\partial t} = D \frac{\partial^2 \rho(x,t)}{\partial x^2} \]

with \( t \in R^+ \) and \( x \in R^s \) (\( s = 1,2, \) or \( 3 \), the dimension of space). The solution of such equations can be written in the form

\[ \rho_t = Y(t)[\rho] \quad \text{with} \quad Y(t) = \exp(-At) \]

and \( \{Y(t) \mid t \in R^+\} \) is a contractive (or "dissipative") semi-group acting on an appropriate Banach space, the space of solutions of the given differential equation.

The microscopic description on the other hand assumes that one has a "large" collection of particles the motion of which is governed by the laws of classical or quantum mechanics. In the latter case, the corresponding equation is the Schroedinger equation:

\[ \frac{\partial \psi(x,t)}{\partial t} = -i \mathcal{H} \psi(x,t) \]

with \( t \in R \) and \( x \in R^{sN} \), \( N \) denoting the number of particles; and where the Hamiltonian \( \mathcal{H} \) is a differential operator acting on \( \mathcal{H} = L^2(R^{sN},dx) \) and typically breaking into the sum of a kinetic energy part:

\[ H_o = \frac{1}{2m} \sum_{n=1}^{N} \frac{\partial^2}{\partial x_n^2} \]

and a potential energy part: