

On higher p -adic regulators

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The comparison between K -theory and étale cohomology makes it possible to exhibit torsion classes in the K -groups of rings of integers of number fields [17]. We show here that it can also be used to produce classes of infinite order. We get this way an explicit construction of the classes of Borel by means of units in cyclotomic extensions.

The construction that is used (products and transfer in K -theory with coefficients) is valid in a more general context. We get by this method a free part in the K -theory (with ℓ -adic coefficients) of a local field. In a forthcoming paper we shall show in this way that the K -theory (with ℓ -adic coefficients, ℓ big enough) of an abelian variety with complex multiplication is of rank at least the order of the zeroes of its zeta function at negative integers.

The construction used here allows us to compare the global situation to the local one. If p is an odd prime, F an abelian number field of degree prime to p , and F_p the product of the completions of F above p . We show that the group $K_{2i-1}(F) \otimes \mathbb{Q}_p$ maps injectively into $K_{2i-1}(F_p) \otimes \mathbb{Q}_p$ as soon as some p -adic L -function is nonzero at the point i . This result, which fits with conjectures of Coates [5], is proved in a way very analogous to the approach to transcendental regulators due to Bloch for K_3 of a cyclotomic field [2].

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