This paper is a continuation of [3], which contains a geometrical characterization of the class of Banach spaces having the unconditionality property for martingale differences (see Definition 1.3). Important information about this class, which we denote by UMD, is contained in the work of Maurey, Pisier, Aldous, and others. For example, if \( B \in \text{UMD} \), then \( B \) is superreflexive [5] but there do exist superreflexive spaces that are not in UMD [6]; if \( 1 < p < \infty \), and the Lebesgue-Bochner space \( L^p_B(0,1) \) has an unconditional basis, then \( B \in \text{UMD} \) [1].

One of the main objects of study in this paper and [3] is the class MT of Banach spaces \( B \) for which \( B \)-valued martingale transforms are well-behaved (see Definition 1.2). The geometrical condition introduced in [3] also characterizes the class MT, information about which is of value in the study of \( B \)-valued stochastic integrals and \( B \)-valued singular integrals. Before recalling the probability background, we shall describe this geometrical condition.

Let \( B \) be a real or complex Banach space. The norm of a point \( x \) in \( B \) will be denoted by \( |x| \). A function \( \zeta : B \times B \to \mathbb{R} \) is symmetric if \( \zeta(x,y) = \zeta(y,x) \) and is biconvex if both \( \zeta(\cdot, y) \) and \( \zeta(x, \cdot) \) are convex on \( B \) for all \( x, y \in B \).

**Definition 1.1.** A Banach space \( B \) is \( \zeta \)-convex if there is a symmetric biconvex function \( \zeta \) on \( B \times B \) such that \( \zeta(0,0) > 0 \) and
Although not obvious from the definition, this class of spaces is large and contains, for example, the Lebesgue spaces, $L^r$ and $L^r(0,1)$, $1 < r < \omega$, as well as all finite dimensional spaces.

One of the main results of [3] can now be stated as follows:

**Theorem 1.1.** For a Banach space $B$, the following statements are equivalent:

(i) $B$ is $\zeta$-convex,

(ii) $B \in \text{UMD},$

(iii) $B \in \text{MT}.$

The existence of a symmetric biconvex function $\zeta$ satisfying (1.1) and $\zeta(0,0) > 0$ has further significance. Any such function determines the norm of $B$ up to equivalence. In fact, let $V$ be the smallest convex set containing those $x$ in $B$ satisfying $\zeta(\alpha x, -\alpha x) > 0$ for all scalars $\alpha$ such that $|\alpha| \leq 1$. Then [3],

$$\|x\| = \inf\{\lambda > 0: x \in \lambda V\}$$

defines a norm on $B$ satisfying

$$|x| \leq \|x\| \leq |x|/\zeta(0,0).$$

Therefore, $\zeta(0,0) \leq 1$ if $B \neq \{0\}$, as we shall always assume, which can also be seen easily from (1.1), and the extreme case $\zeta(0,0) = 1$ implies that $|x| = \|x\|$. The extreme case is also interesting for another reason: If $\zeta(0,0) = 1$, then $B$ is an inner product space as we shall prove in Section 2.

Now let $f = (f_1, f_2, \ldots)$ be a $B$-valued martingale and $g = (g_1, g_2, \ldots)$ the transform of $f$ by a real-valued predictable sequence $v = (v_1, v_2, \ldots)$. That is, there is a probability space $(\Omega, \sigma, \mathbb{P})$ and a nondecreasing sequence $\sigma_0, \sigma_1, \ldots$ of sub-$\sigma$-fields of $\sigma$ such that