Introduction to Holonomic Quantum Fields

by

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The purpose of this paper is to present an introduction to the theory of what we call the holonomic quantum fields; to be specific we give a brief survey to each of our series of short notes entitled "Studies on Holonomic Quantum Fields, I-XVI" [1] as well as of the earlier preprint [2].

We would like to express our heartiest gratitude to Professor M. Sato, who has introduced this subject to us who has been and is working with us with his constant enthusiasm toward science.

The materials treated in [1], [2] are grouped as follows:

(1) Algebraic preliminaries (V and part of I III, Iv, X).

(2) Ising model ([2],I) and the deformation theory of Euclidean Dirac-equation (II, III, IV; VII, VIII, IX).

(3) Riemann-Hilbert problem (VI).

(4) Higher dimensions (XI, XII, XIII, XIV).

(5) Impenetrable bosons (XVI) and double scaling limit of XY model (XV).

A full account for (1), (2), (3) and (5) are found in [3], [4], [5] and [6], respectively.

<u>Field theory of the 2-dimensional Ising model in the scaling limit [2]</u>.

1. Matrix elements of the Ising spin above the critical temperature are calculated in the infinite lattice. The reference states are those in the Fock space of free fermions originally introduced by Onsager.

2. Taking the scaling limit of the Ising spin, we obtain a local field operator $\varphi(x)$ in 2-dimensional space-time. $\varphi(x)$ is expressed as the following normal product of free fermion $\psi(p)^+ = \theta(p^o)\psi(\vec{p})^+ + \theta(-p^o)\psi(-\vec{p}$ $(p = (p^o,\vec{p})$ is on the mass shell $p^2 = (p^o)^2 - (\vec{p})^2 = m^2)$.

$$(1)\qquad \varphi(x) = :\psi(x)\cdot\exp\left(\frac{1}{2}\iint \underline{dp_1 dp_2}\, 2i\frac{p^o - p^{o\prime}}{\vec{p}+\vec{p}\,'+i0}\psi(p_2)^+\psi(p_1)^+ e^{i(p_1+p_2)x}\right):,$$

$$\psi(x) = \int \underline{dp}\ \psi(p)\ e^{ipx},$$

$$\underline{dp} = \frac{1}{2\pi} \frac{d\vec{p}}{2\sqrt{\vec{p}^2+m^2}} .$$

3. The asymptotic fields $\varphi^{in}$, $\varphi^{out}$ for $\varphi(x)$ are calculated. There is no particle production, and the S matrix in the n-particle sector is found to be

$$S = (-1)^{\frac{n(n-1)}{2}} . \tag{2}$$

4. We have checked the generalized unitarity relation for our field operator $\varphi(x)$.

Studies on Holonomic Quantum Fields I.

1. Without recourse to the lattice theory, direct construction of the spin operators $\varphi_F(x)$ $(T \le T_c)$ and $\varphi^F(x) = \varphi(x)$ in (1) $(T \ge T_c)$ in the continnum is presented. Guiding principle is the theory of Clifford group. $\varphi_F(x)$ is introduced as the Clifford group element which induces the following "rotation" $T_{\varphi_F(x)}$ in the orthogonal space $W = \{w(x) = (w_+(x), w_-(x)) \mid \frac{\partial w_\pm}{\partial x^\pm} = \pm m w_\mp\}$ of solutions to free neutral Dirac equation.

$$T_{\varphi_F(x)}(w^+ + w^-) = w^+ - w^- \quad \text{if} \quad w^\pm \text{ belongs to} \tag{3}$$

$$W_x^\pm = \{w \in W \mid w(x') = 0 \quad \text{for} \quad (x - x')^2 < 0,\ \ x'^1 - x^1 \lessgtr 0\}.$$

2. The symplectic (or Bosonic) version of the above construction is also given.

3. The Landau singularities of the n point Green's functions for the above operators are confined to those corresponding to graphs with no internal vertices. The order of the leading singularity is determined.

Studies on Holonomic Quantum Fields II.

1. The following monodromy problem is discussed for solutions