A locally convex algebra $A$ is a real or complex associative algebra, which is a locally convex Hausdorff topological vector space and the multiplication $(x,y) \rightarrow xy$ is a jointly continuous bilinear operation from $A \times A$ to $A$. The topology of a locally convex algebra $A$ can be given by means of a family $\mathcal{\phi} = (\alpha_1, \ldots, \alpha_n)$ of seminorms such that for each index $\alpha$ there is a $\beta \in \mathcal{\phi}$ with

\begin{equation}
|x y|_{\alpha} \leq |x|_{\beta} |y|_{\beta}
\end{equation}

for all $x, y$ in $A$. Without loss of generality it can be assumed that $\mathcal{\phi}$ has the following property: together with a finite set of indices $\{\alpha_1, \ldots, \alpha_n\}$ the (continuous) seminorm

\begin{equation}
|x| = \max\{|x|_{\alpha_1}, \ldots, |x|_{\alpha_n}\}
\end{equation}

is in $\mathcal{\phi}$ too. This property has a technical character and for the purpose of this paper we call it property (m). It was a longstanding problem posed in [4] (see also [5, p 78]), whether in the case when $A$ possesses a unit element $e$ the seminorm in $\mathcal{\phi}$ can be chosen so that

\begin{equation}
|e| = 1
\end{equation}

for all $\alpha$ in $\mathcal{\phi}$. The problem has been recently solved in the affirmative by Fernández and Müller [1] who proved that if $A_0$ is a subalgebra of a locally convex algebra $A$ then any system $\mathcal{\phi}_0$ of seminorms on $A_0$ satisfying (1), having the property (m) and giving the (relative) topology of $A_0$, can be extended to a system $\mathcal{\phi}$ of seminorms on $A$, giving its topology and satisfying (1). More precisely, $\mathcal{\phi}_0$ equals to the family of restrictions to $A$ of the member of $\mathcal{\phi}$. If $A$ has a unit element $e$, then taking as $A_0$ the scalar multiples of the unit $\lambda e$ and setting $|\lambda e| = |\lambda|$, what is a norm on $A_0$, we see that the system $\mathcal{\phi}$ obtained in the theorem of Fernandez and Muller satisfies both (1) and (3).
The purpose of this paper is to modify the proof of the above result, so that it can be applied to a more general class of locally pseudoconvex algebras. While the result cannot be literally extended to this class (see remark at the end of the paper) some its variation is still true and it is strong enough in order to obtain formulas analogous to (1) and (3).

Let \( X \) be a real or complex vector space. A non-negative function \( x \rightarrow ||x|| \) on \( X \) is said to be a \( p \)-homogeneous seminorm, or a seminorm with exponent \( p \), \( 0 < p \leq 1 \), if

(i) \( ||x+y|| \leq ||x|| + ||y|| \)

for all \( x, y \in X \), and

(ii) \( ||\lambda x|| = |\lambda|^p ||x|| \)

for all scalars \( \lambda \) and all elements \( x \) in \( X \).

Note that if \( ||x|| \) is a seminorm of exponent \( p \), and \( q \) is a real number satisfying then \( ||x||^q \) is a seminorm of exponent \( pq \). The relation (i) follows here immediately from the inequality

\[
(1+|\lambda|)^q \leq 1 + |\lambda|^q
\]

what implies immediately

\[
\left( \sum_{i=1}^{n} |\lambda_i| \right)^q \leq \sum_{i=1}^{n} |\lambda_i|^q
\]

for any \( n \)-tuple of scalars \( \lambda_1, \ldots, \lambda_n \).

A locally pseudoconvex space \( X \) (see [2, Chapter 3], or [3,p 4]) can be described as a topological vector Hausdorff space whose topology is given by the means of a family \( \phi(x)=(1.1_{\alpha})_{\alpha \in \alpha} \) of \( p(\alpha) \)-homogeneous seminorms, \( 0 < p(\alpha) \leq 1 \). That means that a net \((x_\alpha)\) tends to zero in \( X \) whenever \( ||x_\alpha||_{\alpha} \to 0 \) for each fixed \( \alpha \) in \( \alpha \).

Relations (i) and (ii) imply that the operations of addition and scalar multiplication are continuous in this topology. A locally pseudoconvex algebra is a real or complex associative algebra which is a locally pseudoconvex space and the multiplication is a jointly continuous bilinear operation. Similarly as in the locally convex case it can be shown that the topology of a locally pseudoconvex algebra \( A \) can be given by means of a family \( \phi(A)=(1.1_{\alpha})_{\alpha \in \alpha} \) of \( p(\alpha) \)-homogeneous seminorms, \( 0 < p(\alpha) \leq 1 \), such that for each \( \alpha \in \alpha \) there is a \( \beta \in \beta \) with